Math 10 Polynomial Factoring Solutions

Factor fully:

1. $15x^5 - 10x^7$
   Pull out the GCF
   $= 5x^5(3 - 2x^2)$

2. $x^2 - 25$
   Difference of squares
   In general, $x^2 - y^2 = (x + y)(x - y)$
   $= (x + 5)(x - 5)$
   Verify: $x^2 - 5x + 5x - 25 = x^2 - 25$

3. $9a^2 - 25$
   $(3a + 5)(3a - 5)$

4. $25a^6 - y^2z^{10}$
   $(5a^3 + yz^5)(5a^3 - yz^5)$

5. $a^2 + 9$
   $(a + 3)(a - 3) = a^2 - 9$
   $(a + 3)(a + 3) = a^2 + 6a + 9$
   $(a - 3)(a - 3) = a^2 - 6a + 9$
   Cannot factor

6. $5x^2 - 45$
   $= 5(x^2 - 9)$
   $= 5(x + 3)(x - 3)$

7. $x^2 - 8x + 15$
   $(x - 3)(x - 5)$

8. $x^2 - 6x - 72$
   $(x + 6)(x - 12)$

9. $3x^2 - 12x + 12$
   $= 3(x^2 - 4x + 4)$
   $= 3(x - 2)^2$

10. $2x^2 + 7x - 4$
    $= (2x - 1)(x + 4)$

11. Factor $4x^2 - 35x + 24$
    $(4x - 3)(x - 8)$

12. Factor $30x^2 + 52x - 48$
    $2(3x - 2)(5x + 12)$

13. Factor $-9(x + 1) + x^2(x + 1)$
    $= (x + 1)(x^2 - 9)$
    $= (x + 1)(x + 3)(x - 3)$

14. Factor $a^2(3x - 1) + 9(1 - 3x)$
    $= a^2(3x - 1) - 9(3x - 1)$
    $= (3x - 1)(a^2 - 9)$
    $= (3x - 1)(a + 3)(a - 3)$

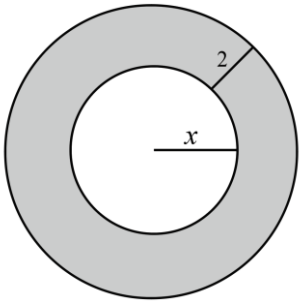
15. $5x^3 - 10x^2 + 3x - 6$
    $= 5x^2(x - 2) + 3(x - 2)$
    $(x - 2)(5x^2 + 3)$

16. $112ab - 16a + 128a^2 - 14b$
    $= 112ab - 14b - 16a + 128a^2$
    $= 14b(8a - 1) - 16a(1 - 8a)$
    $= 14b(8a - 1) + 16a(8a - 1)$
    $= (8a - 1)(14b + 16a)$
    $= 2(8a - 1)(7b + 8a)$

17. $-4x^4y + 12x^3 + x^2y - 3x$
    $= -4x^4y + x^2y + 12x^3 - 3x$
    $= x^2y(-4x^2 + 1) + 3x(4x^2 - 1)$
    $= 3x(4x^2 - 1) - x^2y(4x^2 - 1)$
    $= (4x^2 - 1)(3x - x^2y)$
    $= (2x + 1)(2x - 1)(3x - x^2y)$
    $= x(2x + 1)(2x - 1)(3 - xy)$

Challenge:

18. What is the area of the shaded region below in fully factored form?



$A_{shaded} = A_{big} - A_{small}$
$A = \pi(x + 2)^2 - \pi x^2$
$= \pi[(x + 2)^2 - x^2]$
$= \pi[x^2 + 4x + 4 - x^2]$
$= \pi[4x + 4]$
$= 4\pi(x + 1)$ units²

19. Factor $2(\sin\theta)^2 - 5\sin\theta - 3$
    $(2\sin\theta + 1)(\sin\theta - 3)$

20. Factor $e^{2x} - 25$ ($e \approx 2.718$ is a special constant)
    $(e^x)^2 - 25$
    $= (e^x + 5)(e^x - 5)$

21. $x^2 + kx + 8$. Find the possible values of k such that this trinomial can be factored.
    $(x + 8)(x + 1) \rightarrow k = 9$
    $(x - 8)(x - 1) \rightarrow k = -9$
    $(x + 2)(x + 4) \rightarrow k = 6$
    $(x - 2)(x - 4) \rightarrow k = -6$

22. Factor $x^3 + 1$
    Difference of cubes formula:
    $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
    $(x + 1)(x^2 - x + 1)$

23. Factor $8a^6 - b^3$
    Difference of cubes formula:
    $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
    Let $x = 2a^2$
    $8a^6 - b^3 = (2a^2 - b)(4a^4 + 2a^2b + b^2)$

24. $(x + k)(x + y - 1) = x^2 + xy - 2x - y + 1$.
    Find k
    $k = -1$

25. Factor $x^2 + xy - y - 1$
    Using trial and error:
    $(x - 1)(x + y + 1)$

26. Factor $x^3 - 3x^2 + 4$
    We will learn this factoring technique (Factor Theorem) in Pre-Calculus 12
    $(x - 2)^2(x + 1)$

27. Factor $x^n - y^n$
    See if you can find a pattern using wolframalpha.com
    $(x - y)$ is always a factor