# Math 10 Linear Functions Solutions (DO NOT WRITE ON THIS PAPER)

1. Slope $= \frac{\text{rise}}{?} = \frac{y_2 - y_1}{?}$

   Slope $= \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = m$

2. Points: $(2, 4)$ and $(5, 10)$. Slope?

   $m = \frac{10-4}{5-2} = \frac{6}{3} = 2$

3. $P(4, -2)$ and $Q(-1, 5)$. Slope?

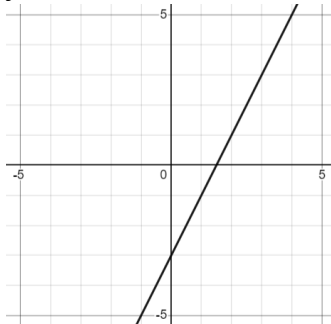   $m = \frac{5-(-2)}{-1-4} = \frac{7}{-5} = -\frac{7}{5}$

4. Points: $(3, 4)$ and $(3, 6)$. Slope?

   Undefined

5. Points: $(1, -2)$ and $\left(2\frac{1}{2}, \frac{3}{4}\right)$. Slope?

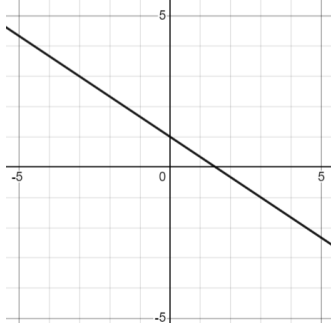   $m = \frac{\frac{3}{4}+2}{\frac{5}{2}-1} = \frac{11}{6}$

6. Points: $A(1, 2)$ and $B(4, a)$. Given the slope of line segment $AB$ is 0, find $a$.

   $0 = \frac{a-2}{4-1}$

   $0 = a - 2$

   $2 = a$
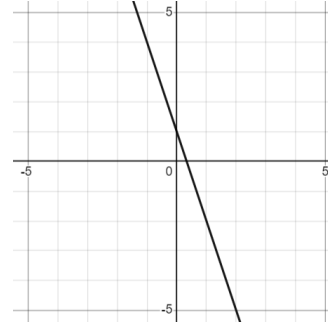
7. Sketch the line:

   a. $y = 2x - 3$
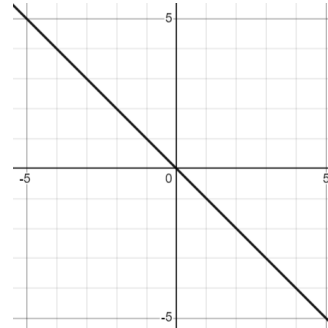
   

   b. $y = \frac{2}{-3}x + 1$
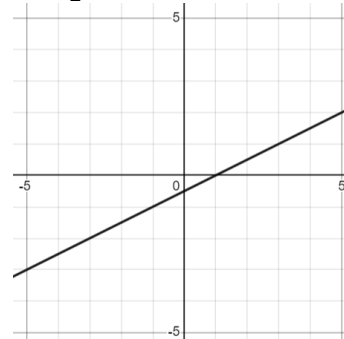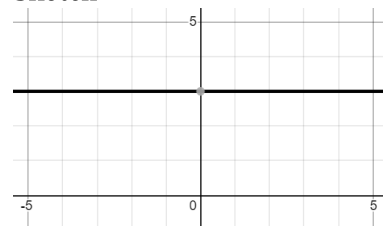
   

c. $y = 1 - 3x$



d. $y = -x$



e. $y = \frac{x}{2} - 0.5$



8. $y = 3$

   a. Find the slope of this line

   0
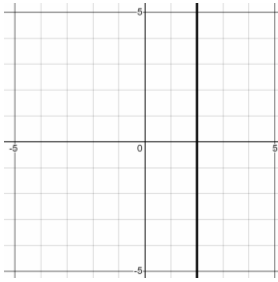
   b. Sketch

   

   c. What quadrants is this line in?

   Quadrants I and II

9. $x = 2$

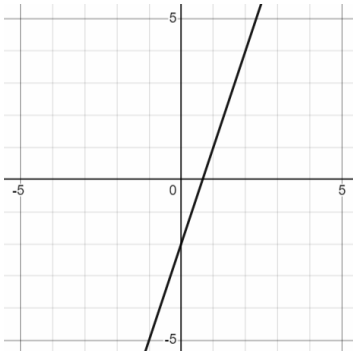   a. Slope?

     Undefined

   b. Sketch



10. $y = 3x - 2$

   a. Slope?

     3

   b. y-intercept?

     $-2$

   c. $(x, y)$ coordinates of the y-intercept?

     $(0, -2)$

   d. x-intercept?

     $0 = 3x - 2$

     $2 = 3x$

     $\frac{2}{3} = x$ or $\left(\frac{2}{3}, 0\right)$

   e. Sketch this line



11. $y = 3x - 2$

   a. Create a table of values for this function

| $x$ | $y = 3x - 2$ |
|-----|--------------|
| 0   | $-2$         |
| 1   | 1            |
| 2   | 4            |
| 3   | 7            |

   b. When $x = 100$, find $y$

     $y = 3(100) - 2 = 298$

   c. When $y = 10$, find $x$

     $10 = 3x - 2$

     $12 = 3x$

     $4 = x$

12. $y = f(x) = 3x + 2$

   a. Is the point $(7, 15)$ on this line?

     When $x = 7$

     $y = 3x + 2 = 3(7) + 2 = 23$

     $23 \neq 15$ thus $(7, 15)$ is not on this line

   b. Is the point $(-5, -13)$ on this line?

     When $x = -5$

     $y = 3x + 2 = 3(-5) + 2 = -15 + 2 = -13$

     $-13 = -13$ thus $(-5, -13)$ is on the line

   c. Evaluate $f(5)$

     $f(5) = 3(5) + 2 = 15 + 2 = 17$

13. Given $h(t) = 2 - 4t$, evaluate $h(-2)$

   $h(-2) = 2 - 4(-2) = 2 + 8 = 10$

14. True of False: $y = -0.\overline{6}x + \frac{1}{5}$ is the same line as $0 = 10x + 15y - 3$

   $y = -\frac{2}{3}x + \frac{1}{5}$

   Multiply by 15
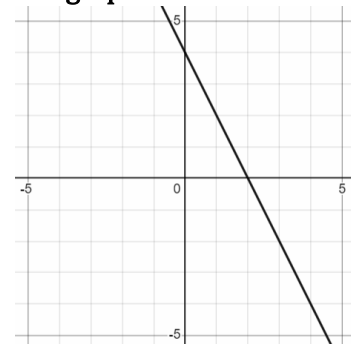
   $15y = -10x + 3$

   $10x + 15y - 3 = 0$

   $0 = 10x + 15y - 3$

   True

   Note: A line written in the form $Ax + By + C = 0$ is in general form
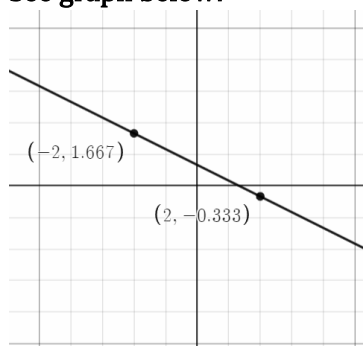
15. What is the equation of the line below?

   a. See graph below:



   $y = -2x + 4$

b. See graph below:



$(-2, 1.667)$
$(2, -0.333)$

$$y = -\frac{1}{2}x + \frac{2}{3}$$

16. A line contains the points $(1, 2)$ and $(5, 0)$
   a. Slope?
   $$m = \frac{0-2}{5-1} = -\frac{2}{4} = -\frac{1}{2}$$

   b. Equation in the form
   $y - y_1 = m(x - x_1)$?
   $y - 2 = -\frac{1}{2}(x - 1)$ or
   $y - 0 = -\frac{1}{2}(x - 5)$

   c. Equation in the form $y = mx + b$?
   We previously found the slope $m = -\frac{1}{2}$
   $y = mx + b$
   $y = -\frac{1}{2}x + b$
   Now substitute either point. We substitute the point $(1, 2)$
   $2 = -\frac{1}{2}(1) + b$
   $2 + \frac{1}{2} = b = \frac{5}{2}$
   Thus $y = -\frac{1}{2}x + \frac{5}{2}$

   d. Intercepts?
   $y = \frac{5}{2}$ and $x = 5$
   i.e. $\left(0, \frac{5}{2}\right)$ and $(5, 0)$
   Find the x-intercept by setting $y = 0$
   $0 = -\frac{1}{2}x + \frac{5}{2}$
   $\frac{1}{2}x = \frac{5}{2}$
   $x = 5$

e. Equation in general form: $Ax + By + C = 0$,
   where the coefficients are integers and $A > 0$
   Previously: $y = -\frac{1}{2}x + \frac{5}{2}$
   Multiply both sides by 2
   $2y = -x + 5$
   $x + 2y - 5 = 0$ (general form line equation)

17. What information is needed to determine the equation of a line?
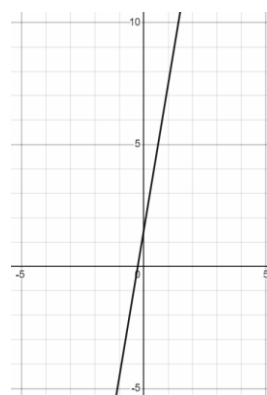   Either a point and the slope or just two points.
   From two points we can determine the slope.

18. $2x + \frac{1}{2} = \frac{y}{3}$
   a. Sketch
   Multiply by 3
   $6x + \frac{3}{2} = y$



   b. Convert to Standard Form: $Ax + By = C$, where $A > 0$
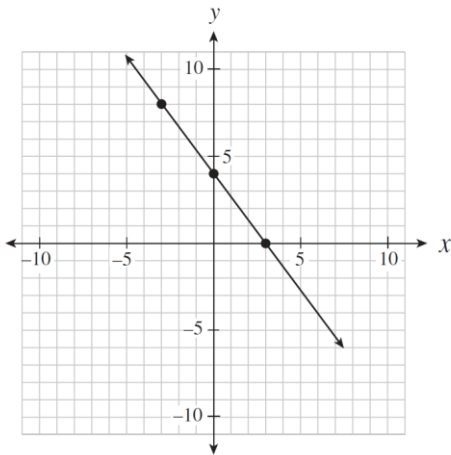   Multiply by 2
   $12x + 3 = 2y$
   $12x - 2y = -3$

   c. Find the intercepts
   y-int: $\frac{3}{2}$
   x-int: $-\frac{1}{4}$

19. See graph below:



Which of the following equations describes the linear relation graphed above?

I. $y = \frac{4}{3}x + 4$

II. $y - 8 = -\frac{4}{3}(x + 3)$

III. $4x + 3y - 12 = 0$

Points on line: $(0, 4)$ and $(3, 0)$

$m = \frac{0-4}{3-0} = -\frac{4}{3}$ (thus eliminate choice I)

III: $3y = -4x + 12 \rightarrow y = -\frac{4}{3}x + 4$

$y - y_1 = m(x - x_1)$
Substitute point $(0, 4)$
$y - 4 = -\frac{4}{3}(x - 0)$
$y = -\frac{4}{3}x + 4$ (which is the same line as III)

II: $y = -\frac{4}{3}x - 4 + 8 = -\frac{4}{3}x + 4$ (which is the same line as well)

Thus II, and II are the same line

20. Consider the pattern $11, 7, 3, -1, ...$
   a. Represent this pattern in the form $y = mx + b$
      $y = -4x + 15$

   b. Find the 1000th number
      $y = -4(1000) + 15 = -3985$

21. Consider the pattern 5, 8 ,11, 14, ...
   a. The variable $f$ represents the figure number. Figure 1 contains the number 5 and figure 2 contains the number 8 and so on. Find the equation $n = af + b$, where $n$ represents the number at a particular figure number.
      $n = 3f + 2$

   b. Find the 100th number
      $n = 3(100) + 2 = 302$

22. See diagram below.
   How many squares are in figure 43?
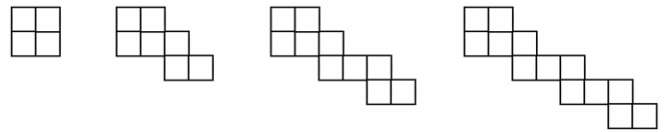


   $y = 3x + 1$
   $y = 3(43) + 1 = 130$

23. How long is the line segment below?



   $8 - 3 = 5$

24. Do the following table of values represent points on a line?
   a. See table below:

   | $x$ | $y$ |
   |-----|-----|
   | 0 | 2 |
   | 1 | 5 |
   | 2 | 8 |
   | 4 | 11 |

   No. The x-values do not increase uniformly by one.

b.  See table below:

| $x$ | $y$ |
|---|---|
| $-2$ | $-4$ |
| $-1$ | $-1$ |
| $0$ | $2$ |
| $2.5$ | $19/2$ |

Yes

$m = \frac{-1-(-4)}{-1-(-2)} = \frac{3}{1} = 3$

Use point $(-2, -4)$

$y + 4 = 3(x + 2)$

$y = 3x + 2$

Substitute $x = 0$

$y = 3(0) + 2 = 2$ (point $(0, 2)$ is on the line)

Substitute $x = 2.5 = \frac{5}{2}$

$y = 3\left(\frac{5}{2}\right) + 2 = \frac{19}{2}$ (point $\left(2.5, \frac{19}{2}\right)$ is on the line)

25. The following table of values represents a line. Find the missing value below:

| $x$ | $y$ |
|---|---|
| $-2$ | $3$ |
| $2$ | $15$ |
| $5$ | $?$ |

$m = \frac{15-3}{2-(-2)} = \frac{12}{4} = 3$

$y - y_1 = m(x - x_1)$

$y - 3 = 3(x + 2)$

$y = 3x + 9 = 3(5) + 9 = 24$

26. Money is a function of time in hours:
    $M(t) = 20t + 50$
    a.  How much do you get paid for working 0 hours?
        $M(0) = 20(0) + 50 = \$50$
        $\$50$

    b.  How much do you get paid if you work for 8 hours?
        $M(8) = 20(8) + 50 = \$210$

    c.  How many hours do you have to work to earn \$280?
        Assume there is no overtime pay.
        $280 = 20t + 50$
        $230 = 20t$
        $11.5 = t$

d.  In the context of this question, what is the domain?
    $t \geq 0$ (also, no human being can work forever)

27. $y = 4x - 3$
    a.  What is the slope of the line that is parallel to this line?
        $m = 4$

    b.  What is the slope of the line that is perpendicular to this line?
        $m_\perp = -\frac{1}{4}$

28. Find the equation of a line that is parallel to $y = 3x - 2$ and:
    a.  goes through the point $(3, 2)$
        $m = 3$
        $y - 2 = 3(x - 3)$
        $y = 3x - 9 + 2$
        $y = 3x - 7$

    b.  has an y-intercept of 4
        Point $(0, 4)$. $m = 3$
        $y - 4 = 3(x - 0)$
        $y = 3x + 4$

    c.  has an x-intercept of 6
        Point $(6, 0)$. $m = 3$
        $y - 0 = 3(x - 6)$
        $y = 3x - 18$

29. Find the equation of a line that is perpendicular to $y = 2x + 1$ and:
    a.  goes through the point $(4, 1)$
        $m_\perp = -\frac{1}{2}$
        $y - 1 = -\frac{1}{2}(x - 4)$
        $y = -\frac{1}{2}x + 2 + 1$
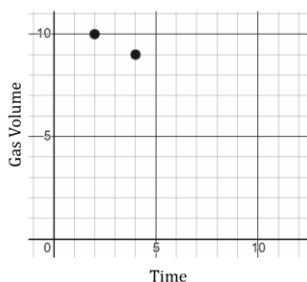        $y = -\frac{1}{2}x + 3$

    b.  goes through the origin
        Point $(0, 0)$.
        $y - 0 = -\frac{1}{2}(x - 0)$
        $y = -\frac{1}{2}x$

30. Draw a line through the two points in the gas – time graph below:



Time

a. When do you run out of gas?
Points: $(2, 10)$ and $(4, 9)$
$m = \frac{9-10}{4-2} = -\frac{1}{2}$
$y - 10 = -\frac{1}{2}(x - 2)$
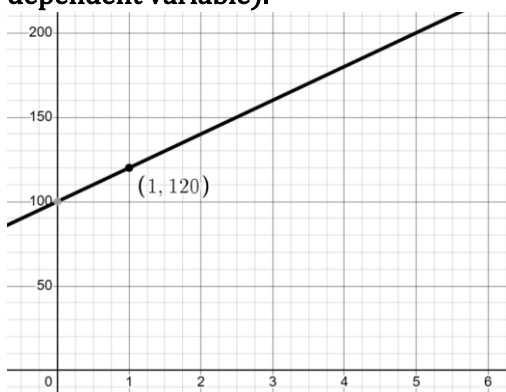$y = -\frac{1}{2}x + 11$
$0 = -\frac{1}{2}x + 11$
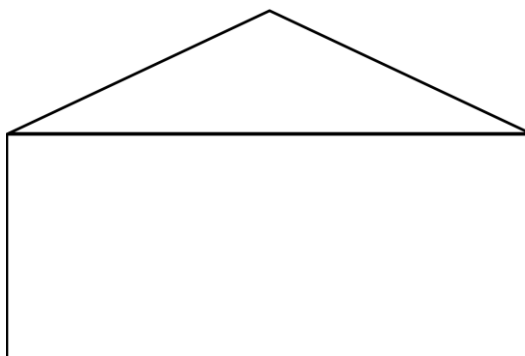$\frac{x}{2} = 11 \rightarrow x = 22$

b. Initial amount of gas?
$y = -\frac{1}{2}x + 11$
$y = -\frac{1}{2}(0) + 11 = 11$

31. You are paid $100 for every day of work, plus $20 per sale for your sales job. Make a graph of your income to sales (with income as your dependent variable).



$(1, 120)$

32. Use a ruler to estimate the positive slope of the house roof below:



Using a ruler, the slope is approximately: $\frac{3}{6.5} = \frac{6}{13} \approx 0.46$ or $0.5$

33. A hot-dog stand owner makes a profit of $100 when he sells 90 hot dogs a day. He has a loss of $30 when he sells 25 hot dogs a day. Model his profit with a line equation.
$(90, 100)$ and $(25, -30)$
$m = \frac{-30-100}{25-90} = \frac{-130}{-65} = 2$
$y - 100 = 2(x - 90)$
$y = 2x - 180 + 100$
$y = 2x - 80$

34. $A\left(\frac{2}{3}, -1\right)$ and $B(k, 3k)$. Given the slope is 2, find $k$.
$2 = \frac{3k+1}{k-\frac{2}{3}}$
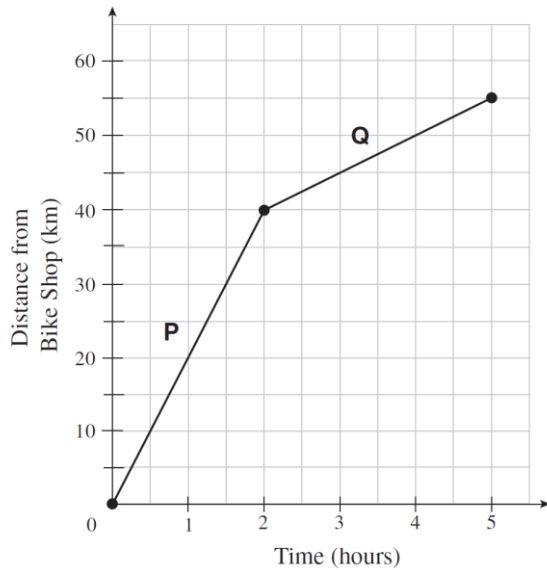Multiply both sides by $k - \frac{2}{3}$
$2\left(k - \frac{2}{3}\right) = 3k + 1$
$2k - \frac{4}{3} = 3k + 1$
$-\frac{4}{3} - \frac{3}{3} = k = -\frac{7}{3}$

35. The graph below models a bicycle's distance from a bike shop over time.
    Calculate the change in the speed of the bike from segment $P$ to segment $Q$.



$$m_1 = \frac{40}{2} = 20$$

$$m_2 = \frac{15}{3} = 5$$

20 kph down to 5 kph means decrease by 15 kph.

36. Given the equation $Ax + By + C = 0$, which of the following conditions must be true for the graph of
    the line to have a positive slope and a positive y-intercept?
    A. $A > 0, B > 0, C > 0$
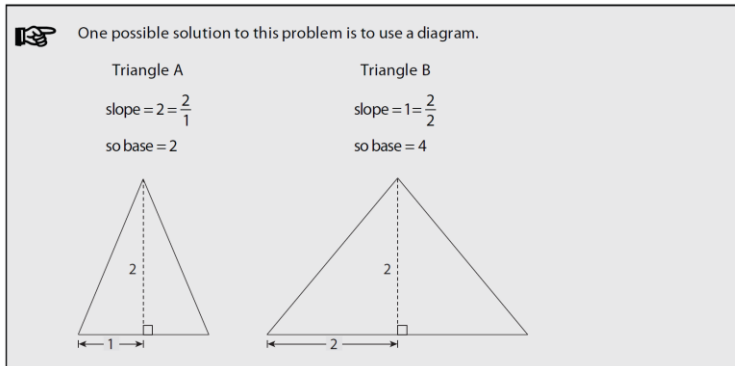    B. $A > 0, B < 0, C > 0$
    C. $A > 0, B > 0, C > 0$
    D. $A > 0, B < 0, C < 0$
    $By = -Ax - C$
    $y = -\frac{A}{B}x - C$
    $A$ and $B$ should have different signs and $C < 0$
    B is the answer

37. Two isosceles triangles have the same height. The slopes of the sides of triangle A are double
    the slopes of the corresponding sides of triangle B. How do the lengths of their bases compare?



Triangle $B$ is 4 times as wide