BC Math 10

Math Practice Problems
Aligned with the BC Math Curriculum
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Topics

Chapter 1: What do you Remember from Math 9?

1. Express $\frac{12}{5}$ as a mixed fraction in the form $a\frac{b}{c}$.
   $2\frac{2}{5}$

2. Evaluate 3^4.
   81

3. Evaluate 5^0 .
   1

4. Evaluate $(-3)^2$.
   9

5. Evaluate -4^2.
   -1×4^2
   -16

6. Simplify $a^4 \times a^3$.
   a^7

7. Simplify $\frac{t^9}{t^5}$.
   t^4

8. $p(p^2)^3$
   $p^1 \cdot p^6 = p^7$

9. $(3x^2)^3$
   $27x^6$

10. $3 - \frac{2}{3}$
    $\frac{9}{3} - \frac{2}{3} = \frac{7}{3}$
    $\frac{7}{3}$ or $2\frac{1}{3}$

11. $-\frac{2}{3} \div \frac{1}{7}$
    $-\frac{2}{3} \times \frac{7}{1} = -\frac{14}{3}$
    $-\frac{2}{3} \times 7 = -\frac{14}{3}$ or $-4\frac{2}{3}$

12. $2\left(\frac{3}{5}\right)^2$
    $2 \times \frac{9}{25} = \frac{18}{25}$

13. $\frac{3}{4} + \frac{1}{2} \times \frac{2}{3}$
    Multiply first!
    Focus on $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$

$\frac{3}{4}+\frac{1}{3}=\frac{13}{12}$
$\frac{9}{12}+\frac{4}{12}=\frac{13}{12}$

14. Expand $2x(x-5)$.
    $2x^2-10x$

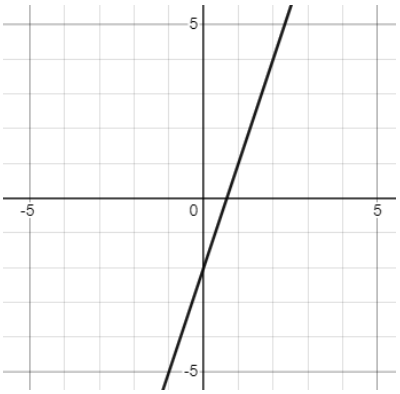
15. Simplify $x^2-x+3+3x^2+5x-1$.
    $4x^2+4x+2$

16. Simplify $(4x-9)-(2x+5)$.
    $4x-9-2x-5$
    $2x-14$

17. Simplify $\left(12x^2-8x\right)\div(4x)$.
    $\frac{12x^2-8x}{4x}=3x-2$
    $\frac{12x^2}{4x}+\frac{-8x}{4x}$
    $3x-2$

18. $y=3x-2$
    a. Find the slope.
       3
    b. Find the y-intercept.
       -2 (by observation)
       Find the y-intercept by setting $x=0$
       $y=3(0)-2=0-2=-2$

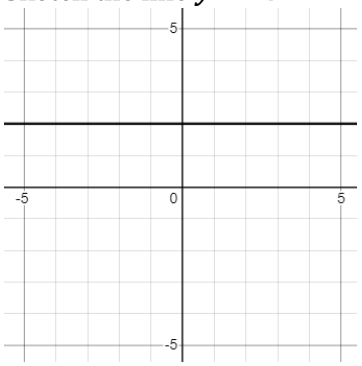
    c. Sketch this line.



    d. When $x=10$, find y.
       $y=3x-2$
       $y=3(10)-2=28$

    e. When $y=13$, find x.
       $y=3x-2$
       $13=3x-2$
       $15=3x$
       $\frac{15}{3}=\frac{3x}{3}$
       $5=x$

19. Sketch the line $y = 2$.



$y = 0x + 2$

20. Solve $\frac{x}{5} = \frac{4}{3}$.

$3x = 5 \times 4$

$3x = 20$

$x = \frac{20}{3}$

21. Solve $-2 = \frac{3}{x}$.

$\frac{-2}{1} = \frac{3}{x}$

$-2x = 3$

$x = \frac{3}{-2}$

$x = -\frac{3}{2}$

22. Solve $3x = 1 - \frac{x+5}{2}$.

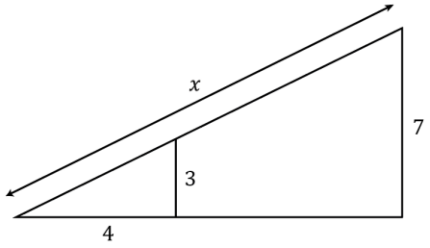
Multiply by 2: $6x = 2 - (x + 5)$

$6x = 2 - x - 5$

$7x = -3$

$x = -\frac{3}{7}$

23. Find x in the diagram below involving right-triangles:



$\frac{3}{5} = \frac{7}{x}$

$3x = 35$

$x = \frac{35}{3}$

Chapter 2: Prime Factorization

Grasping the concept of prime factorization trees enhances your comprehension of the mathematical topic "entire vs. mixed radicals." Determining the least common multiple (LCM) of two or more numbers simplifies the process of adding and subtracting fractions. Recognizing the greatest common factor (GCF) serves as the initial step in mastering factoring techniques.

- Expressing prime factorization of a number using powers
- Identifying the factors of a number
- Includes greatest common factor (GCF) and least common multiple (LCM)
- Strategies include using factor trees and factor pairs

1. Enrichment: List the first four prime numbers.
   $2, 3, 5, 7$

2. Can prime numbers be negative?
   No

3. Find the prime factorization of 27000.
   $2^3 \times 3^3 \times 5^3$

4. What are the factors of 12?
   $1, 2, 3, 4, 6, 12$

5. What are the prime factors of 24?
   $2, 3$

6. Find the GCF and LCM of:
   a. 10 and 15

5	10	15
	2	3

   GCF 5, LCM 30

   b. 8, 12, and 20

4	8	12	20
	2	3	5

   GCF 4, LCM 120

   c. 6, 20, and 30

2	6	20	30
5	3	10	15
3	3	2	3
	1	2	1

   GCF 2, LCM 60

7. Find the GCF and LCM of $15ab^2, 10a^3b^5, 25a^2b^7$.
   GCF: $5ab^2$
   LCM: $150a^3b^7$

8. Find the square root of 900 by finding its prime factorization.
   $900 = 9 \times 100 = 3 \times 3 \times 10 \times 10 = 2 \times 2 \times 3 \times 3 \times 5 \times 5$
   $\sqrt{2 \times 2 \times 3 \times 3 \times 5 \times 5} = \sqrt{2 \times 2} \cdot \sqrt{3 \times 3} \cdot \sqrt{5 \times 5} = 2 \times 3 \times 5 = 30$
   Or $\sqrt{900} = \sqrt{30} \times \sqrt{30} = 30$

9. Find the cube root of 216 by finding its prime factorization.
   $\sqrt[3]{216} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3} = \sqrt[3]{2^3} \times \sqrt[3]{3^3} = 2 \times 3 = 6$

10. Challenge: Why do perfect squares have an odd number of factors?
    Perfect squares have an odd number of factors because their factors form pairs, but the square root pairs with itself, resulting in one unpaired factor. This is reflected in their prime factorization, where all exponents are even, leading to an odd number of factors via the formula $(2a + 1)$ for each prime's exponent.

Chapter 3: Review of BEDMAS, Algebra, and Equations

While algebra is not a formal requirement of the BC Math 10 curriculum, it is included because it provides fundamental skills essential for solving mathematical problems.

1. $209 + 366$
   575

2. $85 - 100$
   -15

3. $\sqrt{9}$
   3

4. $\sqrt{-4}$
   undefined or $2i$

5. $\sqrt[3]{-27}$
   -3

6. 0×1
   0

7. $-3^2 + (-2)^2$
   $-9 + 4 = -5$

8. $(-1)^{123}$
   -1

9. $2 - (-3)$
   $2 + 3 = 5$

10. $-2(-2)^2$
    $-2 \times 4 = -8$

11. Simplify $\frac{40}{1200}$.
    $\frac{4}{120} = \frac{2}{60} = \frac{1}{30}$

12. Simplify $\frac{7500}{250}$.
    $\frac{750}{25}$
    25 fits into 100 four times.
    $4 \times 7 = 28$
    But 25 fits into 50 twice.
    $28 + 2 = 30$

13. Write 0.04 as a simplified fraction.
    $\frac{0.04}{1} = \frac{4}{100} = \frac{1}{25}$

14. Write $-2\left(\frac{4}{-6}\right)$ as a mixed fraction.

$-2\left(\frac{4}{-6}\right) = \frac{8}{6} = \frac{4}{3} = 1\frac{1}{3}$

15. $\frac{5}{4} - \frac{3}{5}$

$\frac{13}{20}$

16. $\frac{4}{6} \times \frac{4}{2}$

$\frac{2}{3} \times 2 = \frac{4}{3}$

17. $\frac{2/3}{3/4}$

$\frac{2}{3} \div \frac{3}{4}$

$\frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$

18. $\frac{8}{3} + 2\frac{1}{2}$

$\frac{8}{3} + \frac{5}{2}$

$\frac{16}{6} + \frac{15}{6} = \frac{31}{6}$

19. $2\frac{2}{3} \div 1\frac{1}{2}$

$\frac{8}{3} \div \frac{3}{2} = \frac{8}{3} \times \frac{2}{3} = \frac{16}{9}$

20. 1.2×0.34

0.408

21. Write $\frac{12346}{5}$

a. in the form $a\frac{b}{c}$.

$2469\frac{1}{5}$

b. as a decimal number.

2469.2

c. as a percent.

$246,920\%$

22. $2 + 3(-2) - 1$

$2 - 6 - 1 = -5$

23. $6 \div 2(1 + 2)$

$3(1 + 2) = 3(3) = 9$

24. $2x - 5x$

$-3x$

25. $\frac{1}{2}x + \frac{x}{3} - x$

$= \frac{3}{6}x + \frac{2x}{6} - \frac{6x}{6}$

$= \frac{3x-4x}{6} = -\frac{x}{6}$

26. $2x - 1 = x + 3$
$x = 4$

27. $\frac{x}{2} + 3 = 3x - \frac{1}{3}$
Multiply both sides by 6
$3x + 18 = 18x - 2$
$20 = 15x$
$\frac{20}{15} = x = \frac{4}{3}$

28. $\frac{x}{5} = \frac{2}{3}$
$3x = 10$
$x = \frac{10}{3}$

29. $5 = \frac{x}{3}$
$x = 15$

30. $-2 = \frac{5}{k}$
$-2k = 5$
$k = -\frac{5}{2}$

31. $\frac{4}{5} = \frac{3}{2x-1}$
Cross multiplying
$4(2x - 1) = 15$
$8x - 4 = 15$
$8x = 19$
$x = \frac{19}{8}$

32. $\frac{x-2}{3} + 2 = \frac{2x+1}{2}$
Multiply by 6
$2(x - 2) + 12 = 3(3x + 1)$
$2x - 4 + 12 = 9x + 3$
$5 = 7x$
$x = \frac{5}{7}$

33. $2(x - 5) = 3(x + 2)$
$2x - 10 = 3x + 6$
$-16 = x$

34. $\frac{2}{3}(1 - 2x) = -\frac{3x+5}{2}$
Multiply both sides by 6
$4(1 - 2x) = -3(3x + 5)$
$4 - 8x = -9x - 15$
$x = -19$

35. $\frac{1-\frac{2}{3}}{\frac{1}{2}+\frac{3}{4}} = 1 \div \frac{1}{x}$

$\frac{1}{3} \div \frac{5}{4} = 1 \times x$

$\frac{1}{3} \times \frac{4}{5} = x$

$x = \frac{4}{15}$

36. Add 6 ft 8 in + 4 ft 6 in in the form x ft and y in.

6 ft + 6 ft = 10 ft

8 in + 6 in = 14 in = 1 ft and 2 in (1 ft = 12 in)

Thus 10 ft + 1 ft + 2 in = 11 ft 2 in

37. A string is cut into three pieces whose lengths form a ratio of 3:5:7. If the string was 105 cm long, how long are the pieces?

There are $3 + 5 + 7 = 15$ parts.

$\frac{105}{15} = 7$

Each part is 7 cm long.

$3 : 5 : 7 \rightarrow 3 \times 7 : 5 \times 7 : 7 \times 7 \rightarrow 21 \text{ cm} : 35 \text{ cm} : 49 \text{ cm}$

Chapter 4: Multiplying Polynomials

Expanding polynomials, a key topic in BC Math 10, involves the reverse process of factoring. Mastering the ability to multiply polynomials and combine like terms accurately is a fundamental and valuable mathematical skill.

- Applying the distributive property between two polynomials, including trinomials
- Connecting the product of binomials with an area model

1. Expand:
   a. $-3x(x-1)$
      $-3x^2+3x$

   b. $2x^2(2-3x+4x^2)$
      $8x^4-6x^3+4x^2$

   c. $(x-3)(x-5)$
      $x^2-8x+15$

   d. $(3x-2)(x-3)$
      $3x^2-11x+6$

   e. $(2x-7)^2$
      $(2x-7)(2x-7)$
      $4x^2-28x+49$

   f. $-3(5-2x)^2$
      $-3(25-20x+4x^2)$
      $=-75+60x-12x^2$
      $=-12x^2+60x-75$

   g. $2(3x-1)(x-2)$
      $2(3x^2-7x+2)$
      $=6x^2-14x+4$

   h. $(x+2)(-2)(x-4)$
      $-2(x+2)(x-4)$
      $=(-2x-4)(x-4)$ or $-2(x^2-2x-8)$
      $=-2x^2+4x+16$

   i. $(x-1)(x^2+x+1)$
      $x^3+x^2+x-x^2-x-1$
      $=x^3-1$

   j. $(a+b)(a^2-ab+b^2)$
      $a^3-a^2b+ab^2+a^2b-ab^2+b^3$
      $=a^3+b^3$

   k. $(x^2+x+1)(1-x-x^2)$
      $=x^2-x^3-x^4+x-x^2-x^3+1-x-x^2$

$= -x^4 - 2x^3 - x^2 + 1$

l. $(x-2)^2(x+1)^2$.

$= (x^2 - 4x + 4)(x^2 + 2x + 1)$

$= x^4 + 2x^3 + x^2$

$\quad -4x^3 - 8x^2 - 4x$

$\quad\quad +4x^2 + 8x + 4$

$= x^4 - 2x^3 - 3x^2 + 4x + 4$

m. $(3x - 3y)^3$

$(3x - 3y)(9x^2 - 18xy + 9y^2)$

$27x^3 - 54x^2y + 27xy^2$

$\quad -27x^2y + 54xy^2 - 27y^3$

$27x^3 - 81x^2y + 81xy^2 - 27y^3$

n. $(2x-1)^4$

$= (2x-1)^2(2x-1)^2$

$= (4x^2 - 4x + 1)(4x^2 - 4x + 1)$

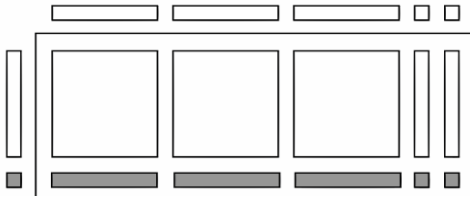
$= 16x^4 - 16x^3 + 4x^2$

$\quad -16x^3 + 16x^2 - 4x$

$\quad\quad 4x^2 - 4x + 1$

$= 16x^4 - 32x^3 + 24x^2 - 8x + 1$

2. Represent the product of the following factors using algebra tiles: $(3x+2)(x-1)$.



3. The length of an edge of a cube is $x - 1$.

a. Find the volume of the cube in the form $ax^3 + bx^2 + cx + d$.

$V = (x-1)^3$

$x^3 - 3x^2 + 3x - 1$ units³

b. Find the area of the cube in the form $ax^2 + bx + c$.

$A = 6 \times (x-1)^2$

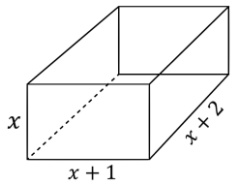
$= 6(x^2 - 2x + 1)$

$= 6x^2 - 12x + 6$ units²

c. Find the volume given $x = 3$.

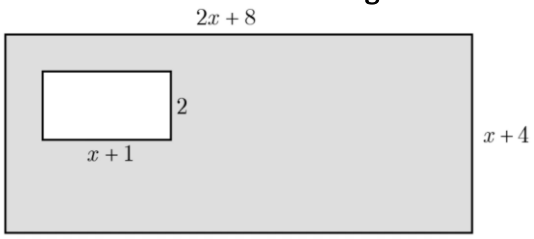
$V = (3)^3 - 3(3)^2 + 3(3) - 1 = 8$ units³

4. Find the surface area of the top of the box below in the form $ax^2 + bx + c$.



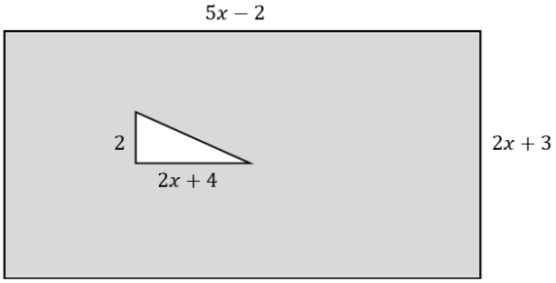
$y = (x+1)(x+2) = x^2 + 3x + 2$

5. Find the area of the shaded region below:



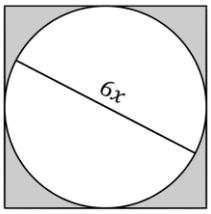
$A = A_{shaded} - A_{white}$
$= (2x + 8)(x + 4) - 2(x + 1)$
$= 2x^2 + 16x + 32 - 2x - 2$
$= 2x^2 + 14x + 30$

6. Find the area of the shaded region below:



$A = (5x - 2)(2x + 3) - \frac{1}{2}(2)(2x + 4)$
$= 10x^2 + 11x - 6 - (2x + 4)$
$= 10x^2 + 9x - 10$ un²

7. Find the area of the shaded region below in the form $ax^2 \pm bx$.

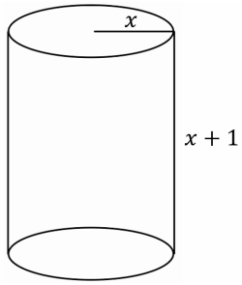


$A = (6x)(6x) - \pi(3x)^2 = 36x^2 - 9\pi x$ un²

8. The diameter of a circle is $2x + 4$.
   a. Area in expanded form?
      $A = \pi(x + 2)^2$
      $= \pi(x^2 + 4x + 4)$
      $= \pi x^2 + 4\pi x + 4\pi$

   b. Circumference in the form $ax + b$?
      $= \pi(2x + 4)$
      $2\pi x + 4\pi$

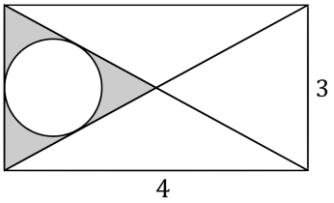
9. See cylinder below:



a. Find the volume in expanded form.
$V = \pi r^2 h = \pi(x^2)(x+1)$
$= \pi(x^3 + x^2)$
$= \pi x^3 + \pi x^2$ un³

b. Find the total surface area including the bottom in expanded form.
$= 2 \text{ circles} + \text{side} = 2\pi r^2 + 2\pi rh$
$= 2\pi x^2 + 2\pi x(x+1)$
$= 2\pi x^2 + 2\pi x^2 + 2\pi x$
$= 4\pi x^2 + 2\pi x$ un²

10. Challenge: Find the area of the shaded region.



Using the Pythagorean Theorem two sides of the shaded triangle has length 2.5
The area of a triangle $A_\Delta = \frac{a+b+c}{2} \times r$ (prove this for fun!)
$A_\Delta = \frac{bh}{2} = \frac{3\times 2}{2} = 3$
Thus $3 = \frac{a+b+c}{2} \times r$
$3 = \frac{3+2.5+2.5}{2} \times r$
Multiply both sides by 2
$6 = 8r$
$r = \frac{6}{8} = \frac{3}{4}$
The area of the circle is $\pi r^2 = \pi\left(\frac{3}{4}\right)^2 = \frac{9\pi}{16}$
Thus $A_{\text{shaded}} = 3 - \frac{9\pi}{16}$

Chapter 5: Polynomial Factoring

Mastering the skill of factoring is a crucial first step in understanding quadratic functions. In this BC Math 10 course, strive to perfect your factoring abilities, as they will remain a vital tool in your mathematical journey through future grades.

- Greatest common factor of a polynomial.
- Simpler cases involving trinomials $y = x^2 + bx + c$ and difference of squares.

1. Factor fully:
   a. $15x^5 - 10x^7$
   $= 5x^5(3 - 2x^2)$

   b. $x^2 - 25$
   Difference of squares
   In general, $x^2 - y^2 = (x + y)(x - y)$
   $= (x + 5)(x - 5)$
   Verify: $x^2 - 5x + 5x - 25 = x^2 - 25$

   c. $9a^2 - 25$
   $(3a + 5)(3a - 5)$

   d. $25a^6 - y^2z^{10}$
   $(5a^3 + yz^5)(5a^3 - yz^5)$

   e. $a^2 + 9$
   $(a + 3)(a - 3) = a^2 - 9$
   $(a + 3)(a + 3) = a^2 + 6a + 9$
   $(a - 3)(a - 3) = a^2 - 6a + 9$
   Cannot factor

   f. $5x^2 - 45$
   $= 5(x^2 - 9)$
   $= 5(x + 3)(x - 3)$

   g. $x^2 - 8x + 15$
   $(x - 3)(x - 5)$

   h. $x^2 - 6x - 72$
   $(x + 6)(x - 12)$

   i. $3x^2 - 12x + 12$
   $3(x^2 - 4x + 4)$
   $3(x - 2)^2$

   j. $2x^2 + 7x - 4$
   $(2x - 1)(x + 4)$

   k. $4x^2 - 35x + 24$
   $(4x - 3)(x - 8)$

l. $30x^2 + 52x - 48$
$2(3x - 2)(5x + 12)$

m. $-9(x + 1) + x^2(x + 1)$
$= (x + 1)(x^2 - 9)$
$= (x + 1)(x + 3)(x - 3)$

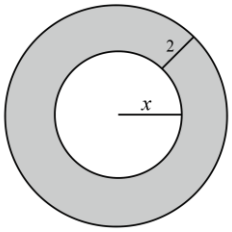
n. $a^2(3x - 1) + 9(1 - 3x)$
$= a^2(3x - 1) - 9(3x - 1)$
$= (3x - 1)(a^2 - 9)$
$= (3x - 1)(a + 3)(a - 3)$

o. $5x^3 - 10x^2 + 3x - 6$
$= 5x^2(x - 2) + 3(x - 2)$
$(x - 2)(5x^2 + 3)$

p. $112ab - 16a + 128a^2 - 14b$
$= 112ab - 14b - 16a + 128a^2$
$= 14b(8a - 1) - 16a(1 - 8a)$
$= 14b(8a - 1) + 16a(8a - 1)$
$= (8a - 1)(14b + 16a)$
$= 2(8a - 1)(7b + 8a)$

q. $-4x^4y + 12x^3 + x^2y - 3x$
$= -4x^4y + x^2y + 12x^3 - 3x$
$= x^2y(-4x^2 + 1) + 3x(4x^2 - 1)$
$= 3x(4x^2 - 1) - x^2y(4x^2 - 1)$
$= (4x^2 - 1)(3x - x^2y)$
$= (2x + 1)(2x - 1)(3x - x^2y)$
$= x(2x + 1)(2x - 1)(3 - xy)$

2. What is the area of the shaded region below in fully factored form?



$A_{shaded} = A_{big} - A_{small}$
$A = \pi(x + 2)^2 - \pi x^2$
$= \pi[(x + 2)^2 - x^2]$
$= \pi[x^2 + 4x + 4 - x^2]$
$= \pi[4x + 4]$
$= 4\pi(x + 1)$ units²

3. Factor $2(\sin\theta)^2 - 5\sin\theta - 3$.
$(2\sin\theta + 1)(\sin\theta - 3)$

4. Factor $e^{2x} - 25$ ($e \approx 2.718$ is a special constant).
$(e^x)^2 - 25$
$= (e^x + 5)(e^x - 5)$

5. $x^2 + kx + 8$. Find the possible values of k such that this trinomial can be factored.
   $(x + 8)(x + 1) \rightarrow k = 9$
   $(x - 8)(x - 1) \rightarrow k = -9$
   $(x + 2)(x + 4) \rightarrow k = 6$
   $(x - 2)(x - 4) \rightarrow k = -6$

6. Challenge:
   a. Factor $x^3 + 1$.
      $(x + 1)(x^2 - x + 1)$

   b. Factor $8a^6 - b^3$.
      Difference of cubes formula: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
      Let $x = 2a^2$
      $8a^6 - b^3 = (2a^2 - b)(4a^4 + 2a^2b + b^2)$

   c. Factor $x^2 + xy - 2x - y + 1$ in the form $(x \pm ?)(? + ? + ?)$.
      Through trial and error: $(x - 1)(x + y - 1)$

   d. $x^3 - 3x^2 + 4$
      We will learn this factoring technique (Factor Theorem) in Pre-Calculus 12
      $(x - 2)^2(x + 1)$

   e. $x^n - y^n$ (See if you can find a pattern using wolframalpha.com)
      See if you can find a pattern using wolframalpha.com

Chapter 6: Operations on Powers

Last year, you explored the fundamentals of exponent laws using whole-number exponents. This year, in BC Math 10, you'll expand your understanding to include negative exponents. Looking ahead to next year, you'll dive into the world of fractional exponents, building on the skills you've already developed.

- Positive and negative exponents.
- Exponent laws.
- Evaluation using order of operations.
- Numerical and variable bases.

1. 2^3
   8

2. $(-5)^3$
   -125

3. -3^2
   -9

4. $(-1)^{100}$
   1

5. $(-1)^{123}$
   -1

6. -1^{666}
   -1

7. $-2(-2)^2$
   $= -2 \times 4 = -8$

8. $-2^2 - (-2)^2$
   $= -4 - 4 = -8$

9. 0^1
   0

10. 1^0
    1

11. π^0
    1

12. 0^0
    Undefined

13. $\left(\frac{2}{3}\right)^2$
    $\frac{4}{9}$

14. 2^{-3}

$= \frac{1}{2^3}$

$= \frac{1}{8}$

15. $\left(\frac{3}{2}\right)^{-2}$

$= \left(\frac{2}{3}\right)^{2} = \frac{4}{9}$

16. $-2(-2)^{-2}$

$= -2 \times \frac{1}{(-2)^2}$

$= -\frac{2}{4}$

$= -\frac{1}{2}$

17. $4(-2)^{-3} \div \frac{1}{2^{-2}}$

$= \frac{4}{(-2)^3} \div 2^2$

$= \frac{4}{-8} \times \frac{1}{4}$

$= -\frac{1}{8}$

18. $a(2a^3 \times 3a^2)$

$6a^6$

19. $\frac{x(x^2)(x^5)}{x^4}$

$= \frac{x^8}{x^4} = x^4$

20. $(2x^3y^2)^3$

$8x^9y^6$

21. $\left(\frac{4x^5}{2x^3}\right)^3$

$= (2x^2)^3$

$= 8x^6$

22. $\left(\frac{3a}{9a^{-2}}\right)^2$

$\left(\frac{a^3}{3}\right)^2 = \frac{a^6}{9}$

23. $\left(\frac{5x^2yz^3}{25x^{-1}y^3}\right)^{-3}$

$= \left(\frac{25x^{-1}y^3}{5x^2yz^3}\right)^3$

$= \left(\frac{5y^2}{x^3z^3}\right)^3$

$= \frac{125y^6}{x^9z^9}$

24. $16^{1-2x} = 2^x$

$(2^4)^{(1-2x)} = 2^x$

$2^{4-8x} = 2^x$

$4 - 8x = x$

$4 = 9x$

$\frac{4}{9} = x$

25. $-3x\left(\frac{2x^5y^{-2}}{x^{-3}y^5}\right)^{-3} \div \frac{1}{x^{-3}y}$

$-3x\left(\frac{x^{-3}y^5}{2x^5y^{-2}}\right)^{3} \times x^{-3}y$

$-3x \cdot \left(\frac{y^7}{2x^8}\right)^{3} \cdot \frac{y}{x^3}$

$-3x \cdot \frac{y^{21}}{8x^{24}} \cdot \frac{y}{x^3}$

$-\frac{3y^{22}}{8x^{26}}$

26. $\frac{125^{3x}}{25^{x-1}} = 625^{2x+1}$

$\frac{(5^3)^{3x}}{(5^2)^{x-1}} = (5^4)^{2x+1}$

$\frac{5^{9x}}{5^{2x-2}} = 5^{8x+4}$

$5^{7x+2} = 5^{8x+4}$

Equating exponents: $7x + 2 = 8x + 4$

$-2 = x$

27. Challenge: $\left[\frac{(-2)^{500}}{-2^{497}}\right]^{-3} \div \left[-\frac{2^{200}}{(-2)^{199}}\right]^{-1} \times \frac{1^0}{\frac{0}{1}-0!\times 1!}$

Hint: $k!$ means k factorial.

$= [-2^3]^{-3} \div [2]^{-1} \times \frac{1}{0-1\times 1}$

$= [-8]^{-3} \div \frac{1}{2} \times -1$

$= \frac{1}{(-8)^3} \times 2 \times -1$

$= \frac{1}{-512} \times -2$

$= \frac{2}{512} = \frac{1}{256}$

28. Challenge: Solve $25^{2x-1} = 6^{x+3}$ using Desmos or a graphing calculator.

$x \approx 1.85$

Chapter 7: Linear Functions

Linear functions are a cornerstone of the BC Math 10 curriculum. While you may already be familiar with foundational concepts like points, slopes, and intercepts from earlier math courses, this year you'll gain a comprehensive understanding of lines as a mathematical concept. Later in the course, you'll also explore the importance of two lines intersecting, deepening your grasp of this essential topic.

- Slope: positive, negative, zero, and undefined
- Types of equations and lines (point-slope, slope-intercept, and general)
- Equations of parallel and perpendicular lines
- Equations of horizontal and vertical lines
- Connections between representations: graphs, tables, equations

1. Slope $= \frac{\text{rise}}{?} = \frac{y_2 - y_1}{?}$.
   Slope $= \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = m$

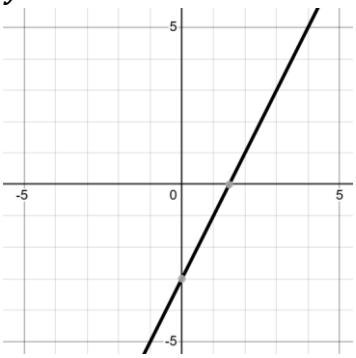
2. Points: $(2, 4)$ and $(5, 10)$. Slope?
   $m = \frac{10-4}{5-2} = \frac{6}{3} = 2$

3. $P(4, -2)$ and $Q(-1, 5)$. Slope?
   $m = \frac{5-(-2)}{-1-4} = \frac{7}{-5} = -\frac{7}{5}$

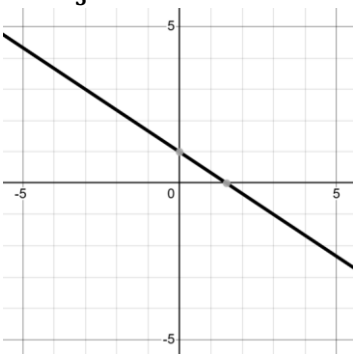
4. Points: $(1, -2)$ and $\left(2\frac{1}{2}, \frac{3}{4}\right)$. Slope?
   $m = \frac{\frac{3}{4}-(-2)}{\frac{5}{2}-1} = \frac{\frac{3}{4}+\frac{8}{4}}{\frac{5}{2}-\frac{2}{2}} = \frac{11}{4} \div \frac{3}{2} = \frac{11}{4} \times \frac{2}{3} = \frac{11}{6}$

5. Points: $A(1, 2)$ and $B(4, a)$. Given the slope of line segment AB is 0, find a.
   $0 = \frac{a-2}{4-1}$
   $0 = \frac{a-2}{3}$
   Multiply by 3
   $0 = a - 2$
   $2 = a$

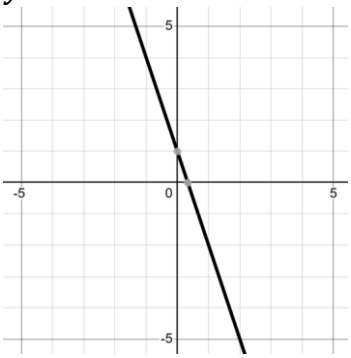
6. Sketch the line:
   a. $y = 2x - 3$



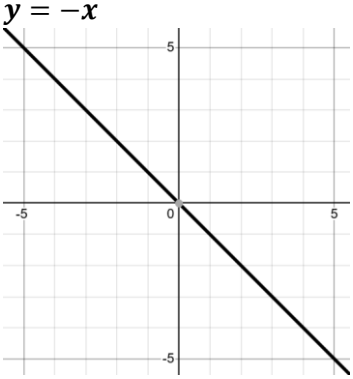
b. $y = \frac{2}{-3}x + 1$



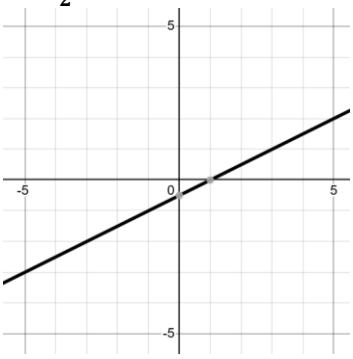
c. $y = 1 - 3x$



d. $y = -x$



e. $y = \frac{x}{2} - 0.5$

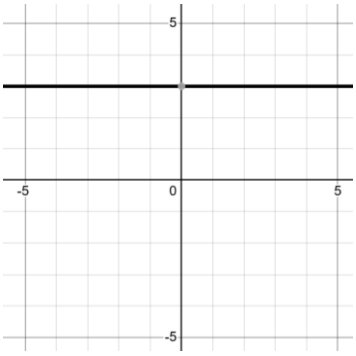


7. $y = 3$

a. Find the slope of this line.

0

b. Sketch

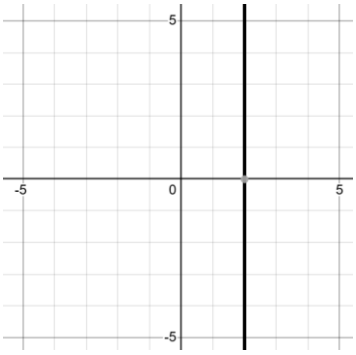


c. What quadrants is this line in?
I, II

8. $x = 2$

a. Slope?
Undefined

b. Sketch



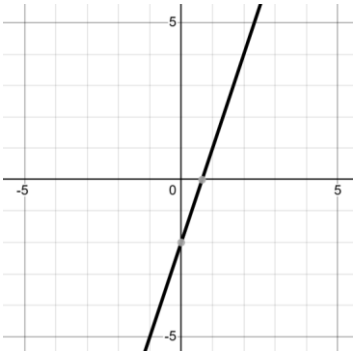
9. $y = 3x - 2$

a. Slope?
3

b. y-intercept?
-2

c. x-intercept?
$0 = 3x - 2$
$2 = 3x$
$\frac{2}{3} = x$

d. Sketch



10. $y = 3x - 2$

a. Create a table of values for this function.

x	$y = 3x - 2$
0	-2
1	1
2	4
3	7

b. When $x = 100$, find y.
$y = 3(100) - 2 = 298$

c. When $y = 10$, find x.
$10 = 3x - 2$
$12 = 3x$
$4 = x$

11. $y = f(x) = 3x + 2$

a. Is the point $(7, 15)$ on this line?
$15? = 3(7) + 2$
$15? = 23$
$LS \neq RS$
No

b. Is the point $(-5, -13)$ on this line?
$-13? = 3(-5) + 2$
$-13? = -13$
$LS = RS$
Yes

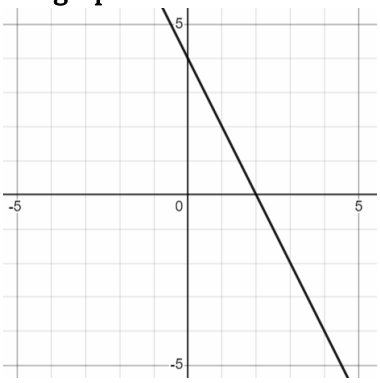
c. Evaluate $f(5)$.
$f(5) = 3(5) + 2 = 17$

12. Given $h(t) = 2 - 4t$, evaluate $h(-2)$.
$h(-2) = 2 - 4(-2) = 2 + 8 = 10$

13. True of False: $y = -0.\overline{6}x + \frac{1}{5}$ is the same line as $0 = 10x + 15y - 3$.
$y = -\frac{2}{3}x + \frac{1}{5}$
Multiply by 15
$15y = -10x + 3$
$10x + 15y - 3 = 0$ which is the same line.

14. What is the equation of the line below in the form $y = mx + b$?

a. See graph below:

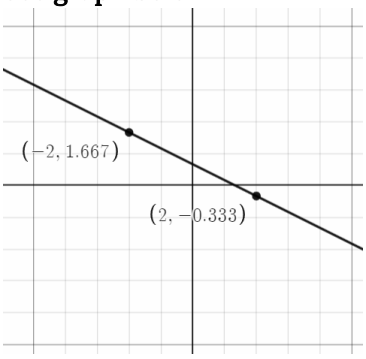


See point $(0, 4)$. The slope is $-\frac{4}{2} = -2$

$y - 4 = -2(x - 0)$

$y = -2x + 4$

b. See graph below:



Points $\left(-2, 1\frac{2}{3}\right)$ and $\left(2, -\frac{1}{3}\right)$

Same as $\left(-2, \frac{5}{3}\right)$ and $\left(2, -\frac{1}{3}\right)$

$m = \frac{-\frac{1}{3}-\frac{5}{3}}{2-(-2)} = -\frac{6}{3} \div 4 = -2 \times \frac{1}{4} = -\frac{1}{2}$

$y - \frac{5}{3} = -\frac{1}{2}(x - (-2))$

$y - \frac{5}{3} = -\frac{1}{2}(x + 2)$

$y = -\frac{1}{2}x - 1 + \frac{5}{3}$

$y = -\frac{1}{2}x + \frac{2}{3}$

15. A line contains the points $(1, 2)$ and $(5, 0)$

a. Slope?

$m = \frac{0-2}{5-1} = -\frac{2}{4} = -\frac{1}{2}$

b. Equation in the form $y - y_1 = m(x - x_1)$?

$y - 2 = -\frac{1}{2}(x - 1)$ or $y - 0 = -\frac{1}{2}(x - 5)$

c. Equation in the form $y = mx + b$?

$y = -\frac{1}{2}x + \frac{1}{2} + \frac{4}{2} = -\frac{1}{2}x + \frac{5}{2}$

d. Intercepts?

y-int: $\frac{5}{2}$

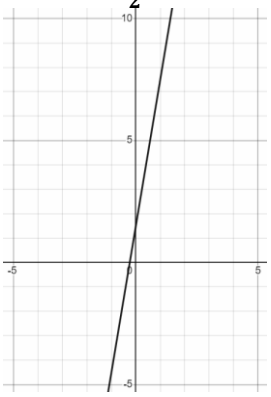
$0 = -\frac{1}{2}x + \frac{5}{2}$
Multiply by 2
$0 = -x + 5$
$x = 5$

e. Equation in general form: $Ax + By + C = 0$, where the coefficients are integers and $A > 0$.
$y = -\frac{1}{2}x + \frac{5}{2}$
Multiply by 2
$2y = -x + 5$
$x + 2y - 5 = 0$

16. What information is needed to determine the equation of a line?
Either a point and the slope or two points.

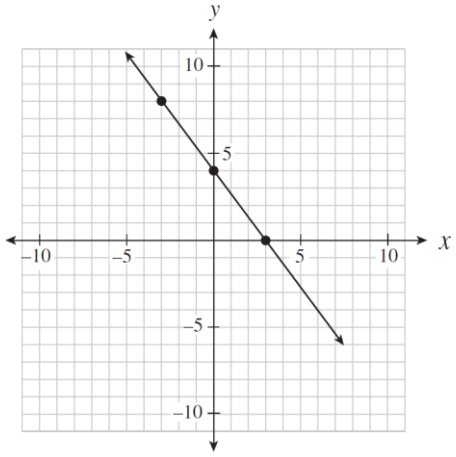
17. $2x + \frac{1}{2} = \frac{y}{3}$

a. Sketch this line.
Multiply by 3
$y = 6x + \frac{3}{2}$



b. Convert to Standard Form: $Ax + By = C$, where $A > 0$.
Multiply by 2
$12x + 3 = 2y$
$12x - 2y = -3$

18. See graph below:



Which of the following equations describes the linear relation graphed above?
I. $y = \frac{4}{3}x + 4$
II. $y - 8 = -\frac{4}{3}(x + 3)$

III. $4x + 3y - 12 = 0$

Eliminate choice I since the slope must be negative.
Choice II has the correct slope and point: $(-3, 8)$.
Choice III: $3y = -4x + 12$
Now divide by 3: $y = -\frac{4}{3}x + 4$ (also correct)

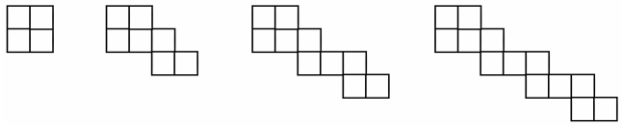
19. Consider the pattern $11, 7, 3, -1, \ldots$
    a. Represent this pattern in the form $y = mx + b$.
    $y = -4x + 15$

    b. Find the 1000^{th} number.
    $y = -4(1000) + 15 = -3985$

20. Consider the pattern 5, 8 ,11, 14, ...
    a. The variable f represents the figure number. Figure 1 contains the number 5 and figure 2 contains the number 8 and so on. Find the equation $n = af + b$, where n represents the number at a particular figure number.
    $n = 3f + 2$

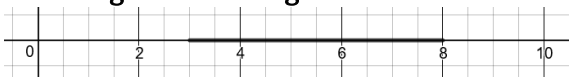
    b. Find the 100^{th} number.
    $n = 3(100) + 2 = 302$

21. See diagram below.
    How many squares are in figure 43?



    $4, 7, 10, \ldots$
    $y = 3x + 1$
    $y = 3(43) + 1 = 130$

22. How long is the line segment below?



    $8 - 3 = 5$

23. Do the following table of values represent points on a line?
    a. See table below:

x	y
0	2
1	5
2	8
4	11

    As x increases by 1, y increases by 3.
    However something goes wrong at $x = 4$.
    When $x = 3$, y should be 11.
    When $x = 4$, y should be 14.
    Not a line.

b. See table below:

x	y
-2	-4
-1	-1
0	2
2.5	$19/2$

Points $(-2, -4)$ and $(-1, -1)$

$m = \frac{-1-(-4)}{-1-(-2)} = \frac{-1+4}{-1+2} = \frac{3}{1} = 3$

Using the point $(-2, -4)$: $y + 4 = 3(x + 2) \rightarrow y = 3x + 2$

Test the point $(0, 2)$

$2? = 3(0) + 2$ (yes)

Test the point $\left(2.5, \frac{19}{2}\right)$

$\frac{19}{2}? = 3\left(\frac{5}{2}\right) + 2$

$RS = \frac{15}{2} + \frac{4}{2} = \frac{19}{2} = LS$ (yes)

Yes, the table represents points on a line.

24. The following table of values represents a line.
Find the missing value below:

x	y
-2	3
2	15
5	$?$

Points: $(-2, 3)$ and $(2, 15)$

$m = \frac{15-3}{2+2} = \frac{12}{4} = 3$

$y - 3 = 3(x + 2) \rightarrow y = 3x + 9$

$y = 3(5) + 9 = 24$

$? = 24$

25. Money is a function of time in hours:
$M(t) = 20t + 50$

a. How much do you get paid for working 0 hours?
$M(0) = 20(0) + 50 = \$50$
$50

b. How much do you get paid if you work for 8 hours?
$M(8) = 20(8) + 50 = \$210$

c. How many hours do you have to work to earn $280? Assume there is no overtime pay.
Assume there is no overtime pay.
$280 = 20t + 50$
$230 = 20t$
$11.5 = t$

d. In the context of this question, what is the domain?
$t \geq 0$ (also, no human being can work forever)

26. $y = 4x - 3$

a. What is the slope of the line that is parallel to this line?
$m = 4$

b. What is the slope of the line that is perpendicular to this line?

$m_{\perp} = -\frac{1}{4}$

27. Find the equation of a line that is parallel to $y = 3x - 2$ and:

a. goes through the point $(3, 2)$.

$m = 3$

$y - 2 = 3(x - 3)$

$y = 3x - 9 + 2$

$y = 3x - 7$

b. has an y-intercept of 4.

Point $(0, 4)$. $m = 3$

$y - 4 = 3(x - 0)$

$y = 3x + 4$

c. has an x-intercept of 6.

Point $(6, 0)$. $m = 3$

$y - 0 = 3(x - 6)$

$y = 3x - 18$

28. Find the equation of a line that is perpendicular to $y = 2x + 1$ and:

a. goes through the point $(4, 1)$.

$m_{\perp} = -\frac{1}{2}$

$y - 1 = -\frac{1}{2}(x - 4)$

$y = -\frac{1}{2}x + 2 + 1$

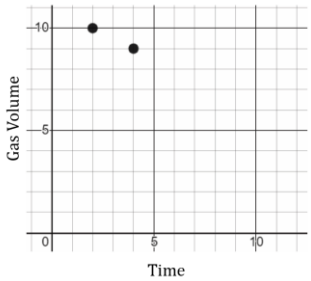
$y = -\frac{1}{2}x + 3$

b. goes through the origin.

$y - 0 = -\frac{1}{2}(x - 0)$

$y = -\frac{1}{2}x$

29. Draw a line through the two points in the gas – time graph below:



a. When do you run out of gas.

Points: $(2, 10)$ and $(4, 9)$

$m = \frac{9-10}{4-2} = -\frac{1}{2}$

$y - 10 = -\frac{1}{2}(x - 2)$

$y = -\frac{1}{2}x + 11$

$0 = -\frac{1}{2}x + 11$

$\frac{x}{2} = 11 \rightarrow x = 22$

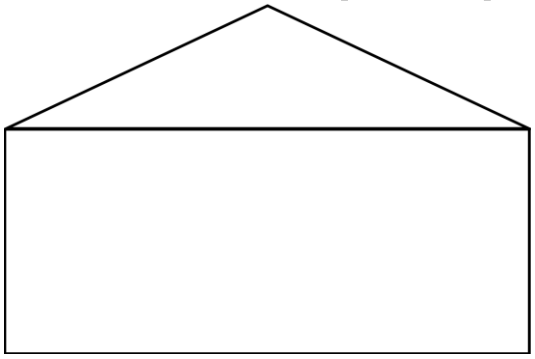
b. Initial amount of gas?

$y = -\frac{1}{2}x + 11$

$y = -\frac{1}{2}(0) + 11 = 11$

30. You are paid \$100 for every day of work, plus \$20 per sale for your sales job.
Make a graph of your income to sales (with income as your dependent variable).

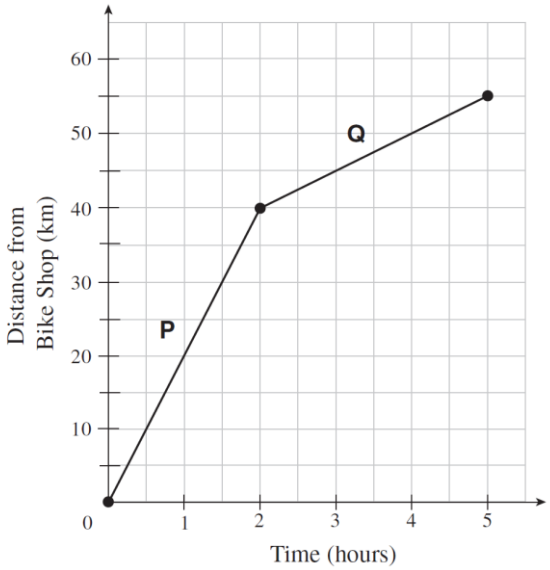
31. Use a ruler to estimate the positive slope of the house roof below:



$m \approx \frac{3.5}{7.5} \approx \frac{7}{15}$ or $\frac{1}{2}$

32. A hot-dog stand owner makes a profit of \$100 when he sells 90 hot dogs a day.
He has a loss of \$30 when he sells 25 hot dogs a day.
Model his profit with a line equation.
Points: $(90, 100)$ and $(25, -30)$
$m = \frac{-30-100}{25-90} = \frac{-130}{-65} = 2$
$y - 100 = 2(x - 90)$
$y = 2x - 80$ or $P = 2h - 80$ (where P represents profit and h represents the number of hot dogs)

33. The graph below models a bicycle's distance from a bike shop over time. Calculate the change in the speed of the bike from segment P to segment Q.



$\textbf{velocity} = \frac{\textbf{distance}}{\textbf{time}}$

$v_1 = \frac{40}{2} = 20$ kph

$v_2 = \frac{15}{3} = 5$ kph

To go from 20 kph to 5 kph the bike decreased by 15 kph.

34. Given the equation $Ax + By + C = 0$, which of the following conditions must be true for the graph of the line to have a positive slope and a positive y-intercept?

A. $A > 0, B > 0, C > 0$

B. $A > 0, B < 0, C > 0$

C. $A > 0, B > 0, C > 0$

D. $A > 0, B < 0, C < 0$

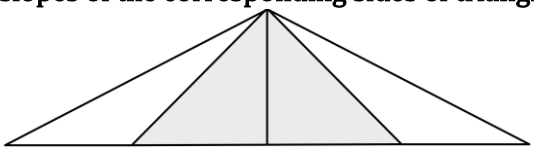
$By = -Ax - C$

Dividing by B

$y = -\frac{A}{B}x - C$

To have a positive slope A, B must have different signs. To have a positive y-intercept, $C < 0$ (since two negatives make a positive).

35. Two isosceles triangles have the same height. The slopes of the sides of triangle A are double the slopes of the corresponding sides of triangle B. How do the lengths of their bases compare?



Notice that the slope of the shaded triangle A is 1. The wide white triangle B has a slope of $\frac{1}{2}$. In other words, triangle A's slope is twice as steep. The base of the wide white triangle B is double that of the shaded base (triangle A).

Chapter 8: Functions and Relations

- Communicating domain and range in both situational and non-situational contexts
- Connecting graphs and context
- Understanding the meaning of a function
- Identifying whether a relation is a function
- Using function notation
- Connecting data, graphs, and situations

1. Domain and Range?
   a. $f(x) = 3x + 2$
      Domain? $x \in \mathbb{R}$ or all real numbers
      Range? $f(x) \in \mathbb{R}$

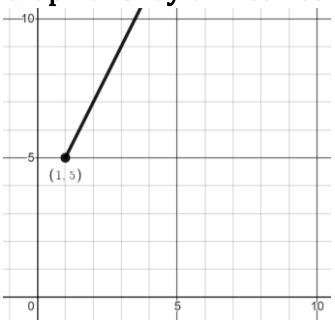
   b. $f(x) = 5$
      Domain? $x \in \mathbb{R}$
      Range? $f(x) = 5 = y$

   c. $x = 3$
      Domain? $x = 3$
      Range? $f(x) \in \mathbb{R}$

2. $f(x) = 2x + 3, x \geq 1$
   a. Domain?
      $x \geq 1$ or $[1, \infty)$

   b. Range?
      $y \geq 5$ or $[5, \infty)$

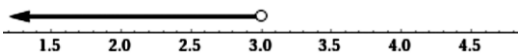
   c. Graph this ray on Desmos using curly brace notation: $y = 2x + 3\ \{x \geq 1\}$.



   d. Evaluate $f(2)$.
      7

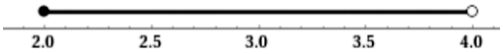
3. Write in interval notation and as a number line:
   a. $y < 3$
      Interval notation: $(-\infty, 3)$

      Number line:

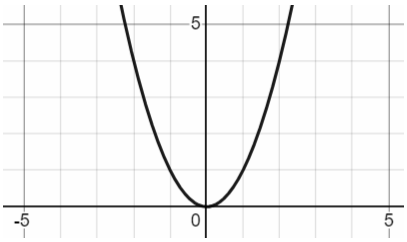


b. $2 \leq x < 4$
Interval notation: $[2, 4)$

Number line:

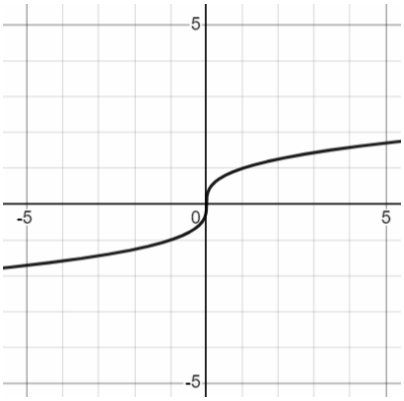


4. Is the following graph a function?
   a. See below:



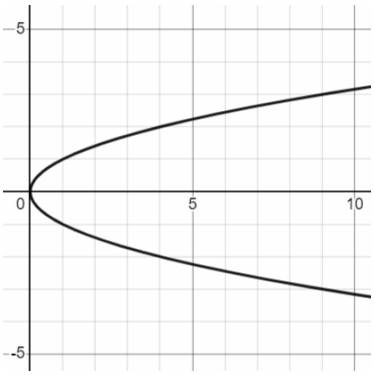
Yes

   b. See below:



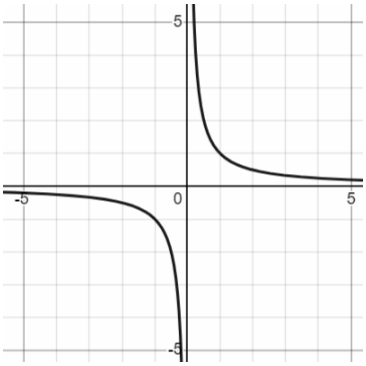
Yes

   c. See below:



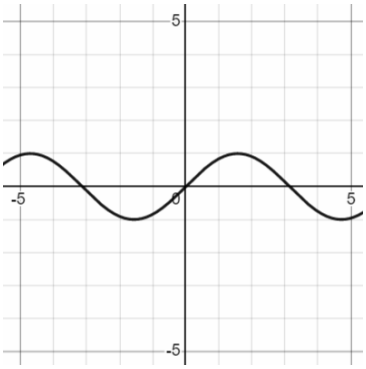
No

d. See below:



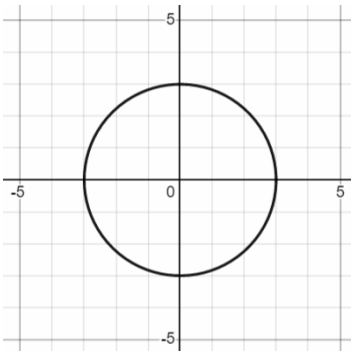
Yes

e. See below:



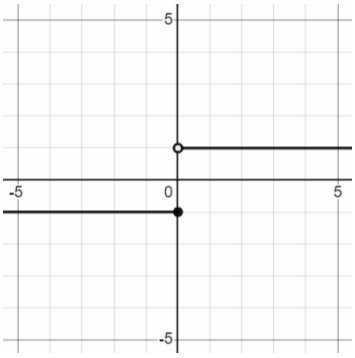
Yes

f. See below:



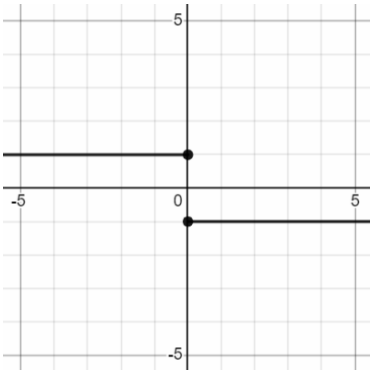
No

g. See below:



Yes

h. See below:



No

5. Evaluate $f(-3)$ given:

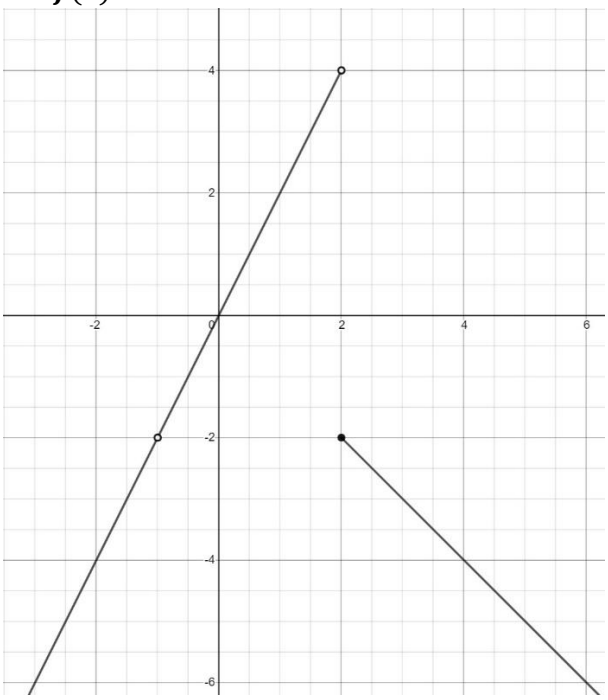
a. $f(x) = x^2$
$(-3)^2 = 9$

b. $f(x) = x^2 - 2x$
$f(-3) = (-3)^2 - 2(-3)$
$= 9 + 6 = 15$

c. $f(x) = 3x^3 - \frac{2^x}{5x}$

$$f(-3) = 3(-3)^3 - \frac{2^{(-3)}}{5(-3)}$$
$$= 3 \times -27 - \frac{1}{8} \div -15$$
$$= -81 - \frac{1}{8} \times -\frac{1}{15}$$
$$-81 + \frac{1}{120} = -\frac{9719}{120}$$

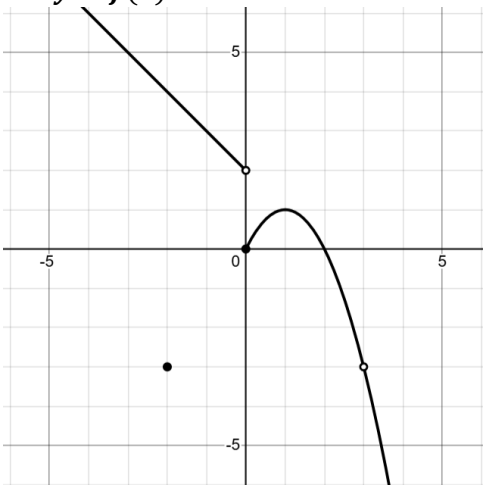
6. See $f(x)$ below:



a. Domain?
$x \neq -1$ ($x \in \mathbb{R}$ is implied)

b. Range?
$y < 4$ (y is defined at $y = -2$)

7. See $y = f(x)$ below:



a. Domain?
$x \neq 3$, ($x \in \mathbb{R}$)

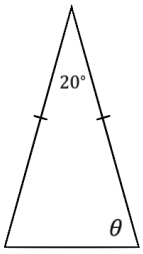
b. Range?
$y > 2$ or $y \leq 1$ or $(-\infty, 1] \cup (2, \infty)$

Chapter 9: Primary Trigonometric Ratios (SOH CAH TOA)

- Sine, cosine, and tangent ratios
- Right-triangle problems: determining missing sides and/or angles using trigonometric ratios and the Pythagorean theorem
- Contexts involving direct and indirect measurement

1. What are the sum of all the angles in a triangle?
   180°

2. Find θ in the diagram below:

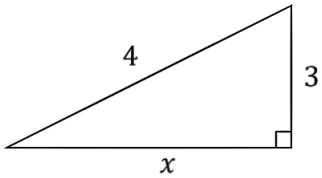


   $2\theta + 20^\circ = 180^\circ$
   $2\theta = 160^\circ$
   $\theta = 80^\circ$

3. Which of the following are Pythagorean Triples?
   I. 3-4-5
   II. 5-12-13
   III. 7-24-25
   IV. 8-15-17
   V. 9-40-41
   VI. 6-8-10
   VII. 50-120-130
   VIII. 1-2-3
   I, II, II, IV, V, VI, VII

4. Evaluate $\sin 30^\circ$ using your calculator in DEGREE mode.
   $\frac{1}{2}$

5. Solve x in the triangle below:



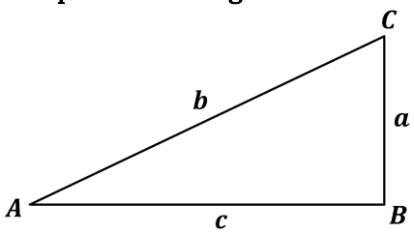
   $x^2 + 3^2 = 4^2$
   $x^2 = 16 - 9$
   $x^2 = 7$
   $x = \sqrt{7}$

6. What does the acronym SOH CAH TOA stand for?
   $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$
   $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

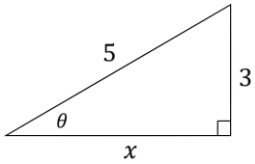
7. Draw the right triangle ABC
   a. Identify $\angle C$
   b. Identify $\angle CAB$
   c. Label the sides with lowercase letters.

   One possible triangle:



   $\angle C$ is the top right angle
   $\angle CAB = \angle A$
   Notice that the lowercase sides are opposite its corresponding angle.

8. See diagram below:



   a. Find x.
   4

   b. Find $\sin\theta$.
   $\sin\theta = \frac{3}{5}$

   c. Find $\cos\theta$.
   $\cos\theta = \frac{4}{5}$

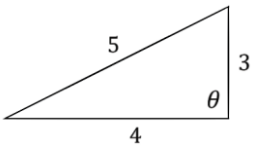
   d. Find $\tan\theta$.
   $\tan\theta = \frac{3}{4}$

   e. Using a calculator $\theta \approx$ _____ degrees.
   $36.9^\circ \approx 37^\circ$

   f. Find the other two missing angles.
   90° and $\approx 53^\circ$

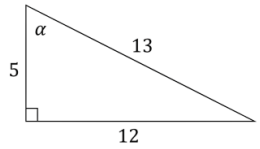
   g. $\theta = \sin^{-1}\left(\frac{a}{b}\right)$ degrees.
   $\theta = \sin^{-1}\left(\frac{3}{5}\right)$

9. True or False: θ in the triangle below must be 90°



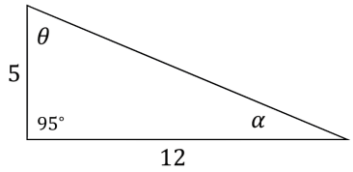
True

10. True or False: $\cos\alpha = \frac{5}{13}$ in the diagram below:



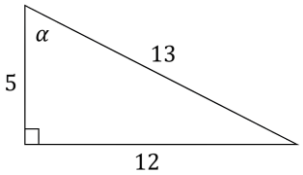
True

11. True or False: $\tan\theta = \frac{12}{5}$ in the diagram below.



False (only works for 90° triangles)

12. See diagram below:

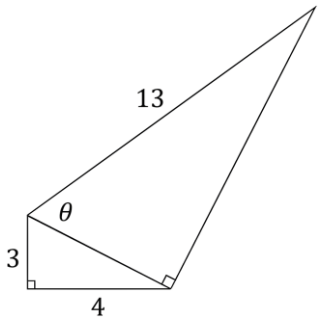


a. Find $\cos\alpha$.
$\cos\alpha = \frac{5}{13}$

b. Perimeter?
$P = 5 + 12 + 13 = 30$ units

c. Area?
$A = \frac{bh}{2} = \frac{(12)(5)}{2} = 30$ units^2

13. See diagram below:

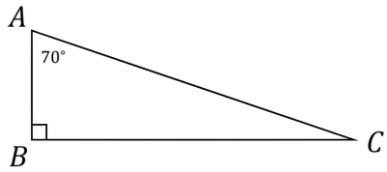


a. Find θ.
$\cos\theta = \frac{5}{13}$
$\theta = \cos^{-1}\left(\frac{5}{13}\right) \approx 67.4^\circ$

b. Label $\alpha = \cos^{-1}\left(\frac{12}{13}\right)$ in the diagram above.
Let the unknown angle in the top right corner be α
$\cos\alpha = \frac{12}{13}$

$$\alpha = \cos^{-1}\left(\frac{12}{13}\right) \approx 22.6^\circ$$

14. See diagram below:

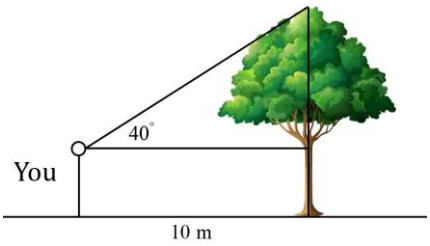


a. Find $\angle C$.
20°

b. Find $\angle ABC$.
90°

c. Given the length of the hypotenuse is 3, find length AB.
$AB = x$
$\cos 70^\circ = \frac{x}{3}$
$x = 3\cos 70^\circ \approx 1.03$

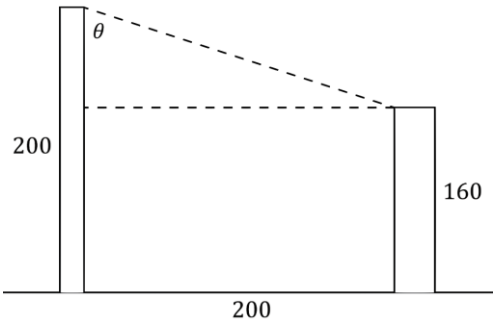
15. Suppose you are 2 m tall. How tall is the tree below?



Let x be the length of the right side of the triangle.
Then $h = x + 2$
Use SOH CAH TOA to find x.
$\tan 40^\circ = \frac{\text{opp}}{\text{adj}} = \frac{x}{10}$
Then $x = 10\tan 40^\circ \approx 8.391$
$h = x + 2 \approx 8.391 + 2 \approx 10.39$ m

16. You look down from a 200 m building to the top of a 160 m building. The buildings are 200 m apart horizontally.

a. Find θ



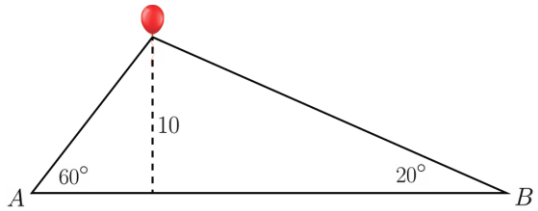
The difference in building height is $200 - 160 = 40$
$\tan\theta = \frac{200}{40} = 5$
$\theta = \tan^{-1}(5) \approx 78.7^\circ$

b. What is your angle of depression?

$90 - \theta \approx 11.3^\circ$

$90 - 78.7 = 11.3$

17. How far apart is person A from person B?



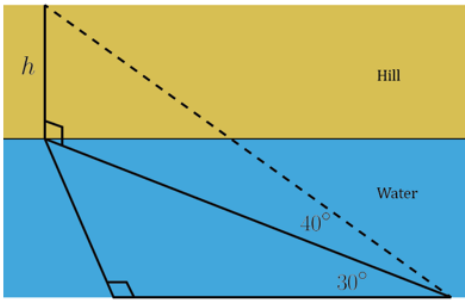
Let the base of the left triangle is x and let the base the right triangle is y

$\tan 60^\circ = \frac{10}{x} \rightarrow x = \frac{10}{\tan 60^\circ}$

$\tan 20^\circ = \frac{10}{y} \rightarrow y = \frac{10}{\tan 20^\circ}$

$x + y \approx 33.2$

18. Find the height of the hill below:



10

First find the hypotenuse of the bottom triangle c:

$\cos 30^\circ = \frac{10}{c} \rightarrow c = \frac{10}{\cos 30^\circ} \approx 11.547$

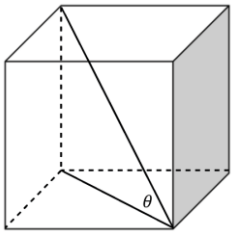
Now focus on the top triangle:

$\tan 40^\circ = \frac{h}{c}$

$c \times \tan 40^\circ = h \approx 9.69$

19. The volume of a cube is 8 m³.

Find the angle formed from the base of the cube from one corner to the furthest other corner.



See the bottom triangle: $2^2 + 2^2 = c^2$

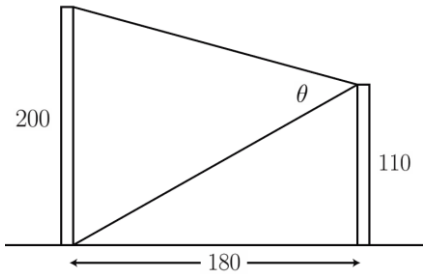
$c^2 = 8 \rightarrow c = \sqrt{8} = 2\sqrt{2}$

Now see the right triangle that contains θ

$\tan \theta = \frac{2}{\sqrt{8}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

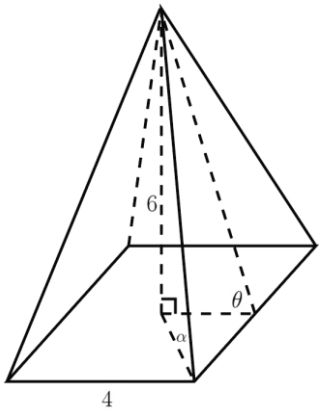
$\theta \approx 35.3^\circ$

20. Find θ in the diagram below:



Draw a horizontal line to split θ and form two right-angled triangles.
The height of the top triangle is $200 - 110 = 90$.
$a + b = \theta$
Let a be the angle in the top triangle and b the angle in the bottom triangle.
$\tan a = \frac{90}{180} = \frac{1}{2}$
$a = \tan^{-1}\left(\frac{1}{2}\right) \approx 26.57°$
$\tan b = \frac{110}{180}$
$b = \tan^{-1}\left(\frac{110}{180}\right) \approx 31.43°$
$\theta \approx 26.57° + 31.43° \approx 58°$

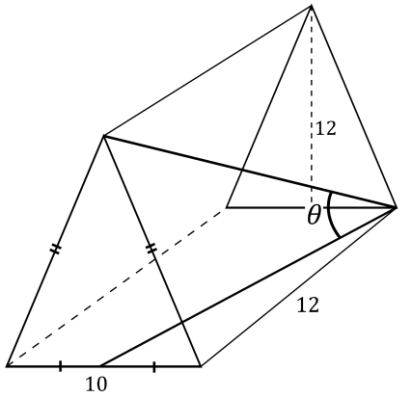
21. The height of the pyramid below is 6. The square base has a side length of 4.



a. Find θ.
Focus on the right triangle containing θ
The base of this triangle is $\frac{4}{2} = 2$ and the height is 6
$\tan\theta = \frac{6}{2} \rightarrow \theta \approx 71.6°$

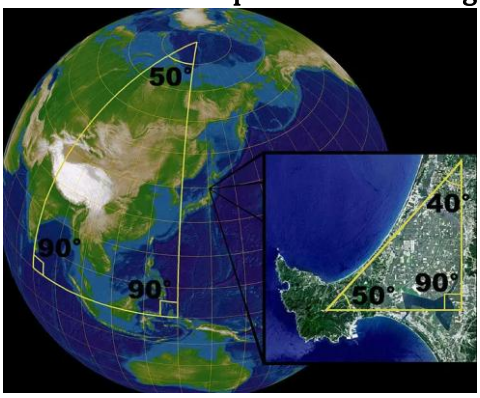
b. Find α.
Now focus on the right triangle containing α
The base of this triangle is c and the height is 6
$c^2 = 2^2 + 2^2$ (Pythagorean Theorem)
$c = \sqrt{8} = 2\sqrt{2}$
$\tan\alpha = \frac{6}{c}$
$\tan\alpha = \frac{6}{2\sqrt{2}} = \frac{3}{\sqrt{2}}$
$\alpha \approx 64.8°$

22. Find θ in the triangular prism below:

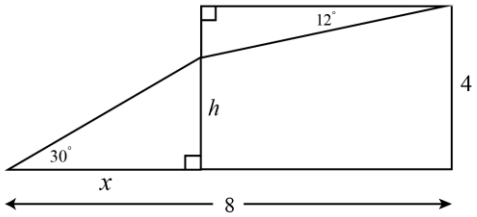


The bottom ray is 13 (Pythagorean Theorem)
The length of the top ray is the length of the side rectangle's hypotenuse.
$c^2 = 36^2 + 40^2 \rightarrow c = \sqrt{2896} \approx 53.814$
$\tan\theta = \frac{12}{13}$
$\theta = \tan^{-1}\left(\frac{12}{13}\right) \approx 42.7^\circ$

23. Fun: Give an example of when the angles in a triangle exceed 180°



24. Challenge: You look up and 30° and see a bird's nest.
Your friend looks down from the window at an angle of depression of 12°.
Solve h in the diagram below:



Focus on the left triangle.
$\tan 30^\circ = \frac{h}{x} \rightarrow h = x\tan 30^\circ$
Now focus on the right triangle.
$\tan 12^\circ = \frac{\text{opp}}{\text{adj}} = \frac{4-h}{8-x}$
$\frac{\tan 12^\circ}{1} = \frac{4-h}{8-x}$
Cross multiply:
$(8-x)\tan 12^\circ = 4 - h$
Divide both sides by $\tan 12^\circ$:
$8 - x = \frac{4-h}{\tan 12^\circ}$

$$\left(8-\frac{4-h}{\tan 12^\circ}\right)=x$$

$$h=x\tan 30^\circ$$
$$h=\left(8-\frac{4-h}{\tan 12^\circ}\right)\tan 30^\circ$$
Using distribution
$$h=8\tan 30^\circ-\frac{\tan 30^\circ(4-h)}{\tan 12^\circ}$$
Multiply both sides $\tan 12^\circ$
$$\tan 12^\circ h=8\tan 30^\circ\times\tan 12^\circ-\tan 30^\circ(4-h)$$
$$\tan 12^\circ h=8\tan 30^\circ\times\tan 12^\circ-4\tan 30^\circ+\tan 30^\circ h$$
Throw $\tan 12^\circ h$ to the right
$$-8\tan 30^\circ\times\tan 12^\circ+4\tan 30^\circ=\tan 30^\circ h-\tan 12^\circ h$$
Factor out the h
$$-8\tan 30^\circ\times\tan 12^\circ+4\tan 30^\circ=h(\tan 30^\circ-\tan 12^\circ)$$
Divide both sides by the coefficient of h
$$\frac{-8\tan 30^\circ\times\tan 12^\circ+4\tan 30^\circ}{\tan 30^\circ-\tan 12^\circ}=h\approx 3.64$$

Chapter 10: Arithmetic Sequences (and Series)

- Applying formal language (common difference, first term, general term) to increasing and decreasing linear patterns
- Connecting to linear relations
- Extension: exploring arithmetic series

1. $2, 5, 8, 11, \ldots$
   a. Find t_1.
   2

   b. Find the common difference d.
   3

   c. Find the 100th number using the t_n arithmetic sequence formula.
   $t_n = t_1 + (n-1)(d)$
   $t_{100} = 2 + (100-1)(3) = 299$

   d. Find the 100th number using your knowledge of lines.
   $y = 3x - 1$
   $y = 3(100) - 1 = 299$

2. Explain why the arithmetic sequence formula works: $t_n = t_1 + (n-1)d$.
   The value of an unknown term is based on the initial term t_1.
   We repeatedly add or subtract the common difference d, $(n-1)$ times.

3. $\frac{1}{2}$ and $\frac{3}{5}$ are the first two terms of an arithmetic sequence. Find the 4th term.
   Find the 4th term.
   The common difference is $\frac{1}{10}$.
   $t_n = t_1 + (n-1)d$
   $t_1 = \frac{1}{2}$
   $n = 4$
   $t_4 = \frac{1}{2} + (4-1)\left(\frac{1}{10}\right)$

4. $45, 40, 35, \ldots, -155, -160$. How many terms are in this sequence?
   $t_n = t_1 + (n-1)d$
   $-160 = 45 + (n-1)(-5)$
   $0 = 205 - 5n + 5$
   $5n = 210$
   $n = 42$

5. The 2nd term of an arithmetic sequence is 7 and the 13th term is -37.
   a. Find t_1.
   $t_2 = 7 \rightarrow t_1 + (2-1)(d) = 7$
   $t_{13} = -37 \rightarrow t_1 + (13-1)(d) = -37$
   We have 2 equations and 2 unknowns so we can solve for d and t_1
   $t_1 + d = 7$
   $t_1 + 12d = -37$
   Subtract equations: $11d = -37 - 7 = -44$

$d = -4$ and $t_1 = 11$.

b. Find t_{10}.
$t_{10} = t_1 + (10-1)(d)$
$= 11 + 9(-4) = 11 - 36 = -25$

6. $2.\overline{6}$ and $1.\overline{3}$ are the third and fourth terms of an arithmetic sequence. Find the first term.
$d = 1\frac{1}{3} - 2\frac{2}{3} = \frac{4}{3} - \frac{8}{3} = -\frac{4}{3}$
$\frac{8}{3} + 2\left(\frac{4}{3}\right) = \frac{16}{3} = 5.\overline{3}$

7. Enrichment: Derive the arithmetic series formula: $S_n = \frac{n}{2}(2t_1 + (n-1)d)$
$S_n = a + (a+d) + (a+2d) + \cdots + (a+(n-1)d)$
$S_n = (a+(n-1)d) + (a+(n-2)d) + \cdots + a$ (same sequence backwards)
$2S_n = [2a+(n-1)d] + [2a+(n-1)d] + \cdots + [2a+(n-1)d]$
Finish finding S_n. (note: $a = t_1$)
$2S_n = n[2a+(n-1)d]$
$S_n = \frac{n}{2}[2a+(n-1)d]$

8. Enrichment: Why are the following formulas equivalent?
$\frac{n}{2}(t_1 + t_n) = \frac{n}{2}(2t_1 + (n-1)d)$
Substitute $t_1 + (n-1)d$ for t_n

9. $2, 5, 8, 11, \ldots$ Find the sum of the first 100 terms.
$S_{100} = \frac{100}{2}\big(2(2) + (100-1)(3)\big)$
$= 50\big(4 + 99(3)\big) = 15050$

10. $11, 8, 5, 2, -1, \ldots, -97, -100$.
Find the sum of this series.
Find n: $t_n = t_1 + (n-1)d$
$-100 = 11 + (n-1)(-3)$
$-111 = -3n + 3$
$3n = 114$
$n = 38$
$S_n = \frac{n}{2}(t_1 + t_n) = \frac{38}{2}\big(11 + (-100)\big) = 19(-89) = -1691$

11. Find the sum of the odd numbers between 100 and 1000.
$S_n = 101 + 103 + 105 + \cdots + 997 + 999$
$t_n = t_1 + (n-1)d$
$999 = 101 + (n-1)(2)$
$898 = 2n - 2$
$900 = 2n$
$n = 450$
$S_{450} = \frac{450}{2}(101 + 999) = 247500$

12. Find the sum of the multiples of 3 between 100 and 200.
$102 + 105 + 108 + \cdots + 198$
$198 = 102 + (n-1)(3)$
$96 = 3n - 3$
$n = 33$

$S_{33} = \frac{33}{2}(102 + 198) = 4950$

13. Sigma Enrichment:
    a. Evaluate $\sum_0^3(2n)$.
    $= 2(0) + 2(1) + 2(2) + 2(3)$
    $= 0 + 2 + 4 + 6$
    $= 12$

    b. Evaluate $\sum_2^5(3x - 2)$.
    $= [3(2) - 2] + [3(3) - 2] + [3(4) - 2] + [3(5) - 2]$
    $= [4] + [7] + [10] + [13]$
    $= 34$

    c. Evaluate $\sum_1^3(x^2 + 2x)$.
    $= \left[(1)^2 + 2(1)\right] + \left[(2)^2 + 2(2)\right] + [(3)^2 + 2(3)]$
    $= [1 + 2] + [4 + 4] + [9 + 6]$
    $= 3 + 8 + 15$
    $= 26$

    d. Evaluate $\sum_1^2 \frac{r}{3r-1}$.
    $= \frac{1}{3(1)-1} + \frac{2}{3(2)-1} = \frac{1}{2} + \frac{2}{5} = \frac{9}{10}$

14. Challenge: $S_2 = 19$ and $S_4 = 50$.
    Find S_{20}, the sum of the first 20 terms of this arithmetic series.
    $S_n = \frac{n}{2}(t_1 + t_n) = \frac{n}{2}(2t_1 + (n-1)d)$
    $19 = \frac{2}{2}[2t_1 + (2-1)d]$ and $50 = \frac{4}{2}[2t_1 + (4-1)d]$
    Equation 1: $19 = 2t_1 + d$
    Equation 2: $50 = 2(2t_1 + 3d)$
    We have two unknowns and two equations so we can solve for d and t_1.
    You may want to study systems of linear equations first.
    $t_1 = 8$ and $d = 3$
    $S_{20} = \frac{20}{2}[2(8) + (20-1)(3)] = 730$

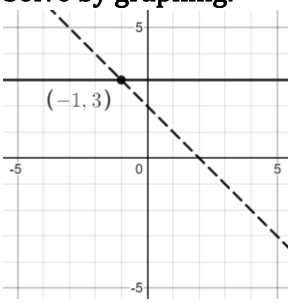
Chapter 11: Systems of Linear Equations

- Solving graphically
- Solving algebraically by inspection, substitution, elimination
- Connecting ordered pair with meaning of an algebraic solution
- Solving problems in situational contexts

1. Confirm the point of intersection:
   a. $x + 2y = 13$ and $3x - y = -11$
      Is $(-1, 7)$ a point of intersection?
      Substitute $(-1, 7)$ into the first equation:
      $(-1) + 2(7) =? 13$
      $13 =? 13$
      Even though the first substitution is successful,
      we must still verify the 2nd equation:
      $3(-1) - (7) =? -11$
      $-10 =? -11$
      $LS \neq RS$, $\therefore (-1, 7)$ is not a point of intersection

   b. $2x - 4 = 4y$ and $x + y = 11$
      Is $(8, 3)$ a point of intersection?
      Substitute $(8, 3)$ into the first equation:
      $2(8) - 4 =? 4(3)$
      $12 =? 12$
      $LS = RS$
      Now substitute $(8, 3)$ into the second equation:
      $(8) + (3) =? 11$
      $11 =? 11$
      $LS = RS$, $\therefore (8, 3)$ is a point of intersection

2. $x + y = 2$ and $y = 3$
   a. Solve by graphing.



      $x = -1$

   b. Solve by substitution.
      $x + (3) = 2$
      $x = -1$

3. $2x + 3y = 12$ and $2y = 8 + 4x$
   a. Find the point of intersection by using substitution.
      Divide equation #2 by 2
      $y = 4 + 2x$
      Substitute into equation #1:
      $2x + 3(4 + 2x) = 12$
      $2x + 12 + 6x = 12$

$8x = 0$
$x = 0$
When $x = 0, 2x + 3y = 12$
$2(0) + 3y = 12$
$3y = 12$
$y = 4 \rightarrow (0, 4)$

b. Find the point of intersection by using elimination.
Multiply equation #1 by 2:
$4x + 6y = 24$
Multiply equation #2 by 3:
$6y = 24 + 12x$
Rearrange
$-12x + 6y = 24$
Subtract the new equations:
$4x - (-12x) = 0$
$16x = 0$
$x = 0$
When $x = 0, 2x + 3y = 12$
$2(0) + 3y = 12$
$3y = 12$
$y = 4 \rightarrow (0, 4)$

4. How many times do the following lines intersect?
   a. $y = 2x + 3$ and $3x - 2y = 1$
   Equation #2: $3x - 1 = 2y$
   Divide by 2
   $\frac{3}{2}x - \frac{1}{2} = y$
   The lines have different slopes, so the lines intersect once.

   b. $4x = 2y - 12$ and $2x + 6 = y$
   Divide equation 1 by 2:
   $2x = y - 6$
   $2x + 6 = y$
   Equation 1 and 2 are identical lines so there are infinite solutions (the lines are coincidental)

   c. $4x + 3 = 2y$ and $y = 2x + 5$
   Divide equation 1 by 2:
   $2x + \frac{3}{2} = y$
   Both lines have the same slope by different y-intercepts therefore the lines do not intersect.

   d. $x = -2$ and $4 - 2x = y$
   $x = -2$ is a vertical line.
   The second line has a slope of -2 so the lines intersect once.

   e. $6x + 16y = 28$ and $\frac{3}{4}x + 2y = 3$
   Divide Equation 1 by 2: $3x + 8y = 14$
   Multiply Equation 2 by 4: $3x + 8y = 24$
   Subtract these equations (bottom equation minus the top): $0 = 10$ which does not make sense. No solution.

   f. $y = 3$ and $y = x^2 - (-2x^2 - 3) - 3x^2$
   Equation 2: $y = x^2 + 2x^2 + 3 - 3x^2 = 3$
   $y = 3$ and $y = 3$ are the same line. These two graphs are coincidental.

5. Systems of Equations word problems:
   a. The difference of two numbers is 3. Their sum is 27. Find the numbers.
   $x - y = 3$ [1]
   $x + y = 27$ [2]
   Adding $[1] + [2]$
   $2x = 30 \rightarrow x = 15$
   When $x = 15$, from [2], $y = 27 - x = 27 - 15 = 12$.

   b. 3 nuts and 5 bolts weigh 16 ounces whereas 2 nuts and 4 bolts weigh 12 ounces. How heavy is a single bolt?
   $3n + 5b = 16$ [1]
   $2n + 4b = 12$ [2]
   $[1] \times 2$ and $[2] \times 3$
   $6n + 10b = 32$ [1a]
   $6n + 12b = 36$ [2a]
   $[2a] - [1a]$
   $2b = 4 \rightarrow b = 2$

   c. Your boat travels 50 km downstream and the trip downstream takes 4 hours. You return to your starting point by traveling upstream for 20 hours against the current. What is the speed of the boat in still water?
   $d = vt$
   Downstream: $50 = (v + c)(4)$ [1a]
   $50 = 4v + 4c$ [1b]

   Upstream: $50 = (v - c)(20)$ [2a]
   $50 = 20v - 20c$ [2b]

   Eliminate c by multiplying [1b] by 5:
   $250 = 20v + 20c$ [1c]

   Now add equations: $[2b] + [1c]$:
   $300 = 40v \rightarrow v = \frac{30}{4} = \frac{15}{2} = 7.5$ kph

   Note: We can substitute $v = \frac{15}{2}$ into [1b]: $50 = 4\left(\frac{15}{2}\right) + 4c \rightarrow 4c = 20 \rightarrow c = 5$
   Indeed, $d = (v + c)(4) = (7.5 + 5)(4) = 50$

   d. The cost to insure jewelry is a fixed amount plus a percentage of the value of the jewelry. It costs \$32 to insure \$1000 worth of jewelry or \$44.50 to insure \$3500 worth of jewelry. What is the fixed amount to insure jewelry?
   Let f be the fixed amount to insure jewellery.
   Let p be the percentage of the value of the jewellery.
   $32 = f + 1000p$
   $44.50 = f + 3500p$
   Equation 2 minus equation 1:
   $2500p = 12.50$
   $p = 0.005$
   Substitute $p = 0.005$ into equation 1:
   $32 - 1000(0.005) = f$
   Thus $f = 27$. The fixed cost is 27.

e. How many ounces of 20% hydrochloric acid solution and 70% hydrochloric acid solution must be mixed to obtain 20 ounces of 50% hydrochloric acid solution?
Let x be the ounces of the weaker acid.
Let y be the ounces of the stronger acid.

$x + y = 20$
Then $y = 20 - x$
$(x)(20\%) + (20 - x)(70\%) = (20)(50\%)$
$0.20x + (20 - x)(0.70) = 20(0.50)$
Multiply by 100
$20x + 70(20 - x) = 20(50)$
$20x + 1400 - 70x = 1000$
$400 = 50x$
$8 = x$
Substitute $x = 8$ into equation 1: $(8) + y = 20$
$y = 12$
To make the right mix we need 8 ounces of the weaker acid
and 12 ounces of the stronger acid.

Chapter 12: Types of Income

- Income tax and other deductions
- Gross and net pay

1. What's the difference between being paid by a salary vs. by an hourly wage?
   Salary: paid a fixed amount in a year (regardless of "overtime" work)
   Hourly wage: paid per hour of work.

2. You work 8 hours a day 5 days a week and make $20 per hour.
   a. How much will your gross pay be this month (assume 4 weeks)?
      Each day: $8 \times \$20 = \$160$
      $\$160 \times 5 \text{ days} \times 4 \text{ weeks} = \$3200$

   b. Now assume you work 10 hours a day. In BC you are paid 1.5 times your pay if your work beyond 8 hours and double pay after 12 hours. How much will your gross pay be this month? Assume that there are 4 weeks in a month.
      Each day: $8 \times \$20 + 2 \times \$30 = \$220$ per day. $220 \times 5 \times 4 = \$4400$ this month

3. Your net income is less than your gross income because of which of the following?
   A. Income Tax (paid to the government at both the Federal and Provincial levels)
   B. Canada Pension Plan (CPP)
   C. Employment Insurance (EI)
   D. All of the above
      D

4. In BC, what percentage of your net income is being deducted?
   (including Federal and Provincial tax)
   (See WealthSimple income tax calculator)
   a. If your annual income is $40,000
      $\approx 18\%$

   b. If your annual income is $80,000
      $\approx 24\%$

   c. If your annual income is $160,000
      $\approx 31\%$

   d. If your annual income is $1,000,000
      $\approx 49\%$

5. Suppose your family gross annual income is $90,000. If the federal and provincial income tax is 25% what is your net monthly income?
   $90,000 \times .75 = 67,500$
   $63,000 \div 12 = \$5625$

6. As a car salesperson how much would you have to sell to match (and possibly surpass) a $60,000 annual salary if you earn $20,000
   plus 5% on the sale of each car you sell?
   Let x be the value of car sales
   $20\,000 + 0.05x = 60\,000$
   $0.05x = 40\,000$

$x = \$800\,000$ in sales

7. True or False:
    a. If you show up for work as scheduled but are sent home because you are no longer needed you must be paid a for a minimum number of hours of work.
    True

    b. In BC you must have a minimum of 8 hours of rest between shifts.
    True

    c. Whether you're 9 or 90, age has no effect on your requirement to file a tax return.
    True

    d. Keeping your receipts is important because taxpayers can receive tax refunds after filing their annual taxes.
    True

    e. If you rent out your basement suite, you can deduct a portion of your utility bills to reduce your taxes.
    True

    f. If you run your business from home, you can deduct a percentage of home expenses as home office expenses on your tax return.
    True

    g. You no longer must file an income tax refund when you retire from work at age 65.
    False

    h. Investment professionals (who charge a 1-2% Management Expense Ratio fee) usually beat the stock market.
    False

    i. You can receive tax deductions if you use your home as an office.
    True

    j. You can receive tax deductions for your home if you have tenants.
    True

    k. Some donations are tax-deductible.
    True

8. There are 3 types of income. Define:
    a. Active income
    You actively work to earn money.

    b. Passive income
    You earn money with little to no effort for it to keep coming.

    c. Portfolio income
    Money that comes from interest on your investments.

9. Explain the following types of income revenue streams:

a. AdSense
Advertisements on your YouTube videos or website. You are typically paid a very small amount of money per 1000 views.

b. Product sales / merchandise
ex. sell an online course, e-book, t-shirt, etc. (ex. using an ecommerce platform such as Shopify).

c. Brand deals
If your audience is large enough, an advertiser may pay you money for you to promote their products or services.

d. Licensing
You are paid for photos or videos (or music for recording artists) that can be used for commercial use.

e. Affiliate
Ex. you are paid for your link to an Amazon product. Some people make a living comparing products.

f. Crowd funding (ex. Patreon or GoFundMe)
People voluntarily support you with a monthly subscription or donation. People will be more willing to support you financially if you provide more direct access to your audience. This is like a product sale, but members support others voluntarily.

g. Private consulting
Private training services can be billed if you are an expert in a particular field.

h. Group live events
Group live events is an extension of private consulting. People will pay to see you in person if you are an expert. For example, companies have a budget for employee professional development. If there is enough demand, you can choose a larger venue to share your expertise with the world!

See: https://www.youtube.com/watch?v=hhnXSsypH7s

10. In the future, if your small business is doing "well," why would you consider becoming "incorporated"?
You can take advantage of additional tax savings and protect your wealth from liability.

11. People like the idea of owning their own business because of greater income potential and being able to set their own hours. When does it make sense to work for someone else?
When you are still learning the business. You have not yet built up your customer base. When you like the idea of separating work life from home life. When you realize that running a successful small business is difficult and requires sacrifice.

12. Canadians are encouraged to invest money into the following accounts: RRSP, TFSA, RESP, and FHSA. What are these for?
RRSP: Registered Retirement Savings Plan (save for retirement).
Helps you receive a larger tax-refund.
TFSA: Tax-Free Savings Account (build savings, tax-free).
Build up your RRSP and TFSA nest egg to generate passive income in your retirement years.

RESP: Registered Education Savings Plan (we are incentivized to make contributions for our children's education)
FHSA: Helps you save up for your First Home. You can also transfer your FHSA to an RRSP.

13. According to https://www.getsmarteraboutmoney.ca/calculators/compound-interest-calculator/ how much wealth can you accumulate if you invest one Starbucks drink a day $7 for 50 years?
    a. at a rate of 10%?
       About $3.7 million
    b. at a rate of 7.5%? (with 2.5% in MER management fees)
       About $1.4 million (banks become rich by skimming off your investments!)
       The moral of the story is to pay less fees (ex. The QQQM index fund has a low MER of 0.2%)

14. Suppose you borrow $20 billion dollars:
    a. How much investment income would you make in one year if the stock market grows by 10%?
       $2 billion
    b. Explain how you can lose $20 billion in two days.
       As a multibillionaire you borrow another $100 billion to invest in Dogecoin. The stock crashes to half it's value. $\$20$ billion + $\$100$ billion = $\$120$ billion goes down to $\$60$ billion in market value. But you still owe the bank $100 billion. You now have a net worth of negative $40 billion!

15. What are two keys to being financially wealthy?
    Make more money (while reducing risk).
    Spend less so you can invest more.

Chapter 13: Core Math 10 Review

1. Write the prime factorization of 840.
   $2^3 \times 3 \times 5 \times 7$

2. What are the factors of the number 8?
   $1, 2, 4, 8$

3. Find the GCF and LCM of 4 and 20.
   GCF: 4
   LCM: 20

4. Solve $3x - 2 = 7x + 3$.
   $-5 = 4x$
   $x = -\frac{5}{4}$

5. $\frac{x}{5} = \frac{7}{2}$
   $2x = 35$
   $x = \frac{35}{2}$

6. $\frac{3}{7} = \frac{5}{x}$
   $3x = 35$
   $x = \frac{35}{3}$

7. $b^3 \times b \times b^5$
   b^9

8. $(x^2)^5$
   x^{10}

9. $\frac{w^{20}}{w^7}$
   w^{13}

10. $\frac{p^4}{p^{-3}}$
    p^7

11. $(5x^3)^2$
    $25x^6$

12. 3^{-3}
    $\frac{1}{3^3} = \frac{1}{27}$

13. $\left(\frac{4}{5}\right)^{-2}$
    $\left(\frac{5}{4}\right)^2 = \frac{25}{16}$

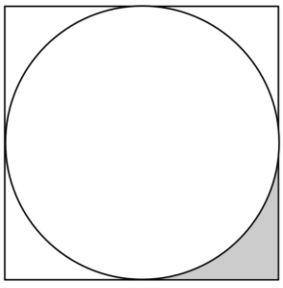
14. $\left(\frac{9x^4}{6x}\right)^{-2}$
    $\left(\frac{6x}{9x^4}\right)^2 = \left(\frac{2}{3x^3}\right)^2 = \frac{4}{9x^6}$

15. $3^2 - (-2)^2$
    $9 - 4 = 5$

16. Expand $(x-5)^2$.
    $x^2 - 10x + 25$

17. Expand $-3(3x-2)^2$.
    $-3(9x^2 - 12x + 4)$
    $-27x^2 + 36x - 12$

18. What is the area of the shaded region below?
    Answer in the form $a \pm b$.



$A_{\text{shaded}} = \frac{(4)(4) - \pi(2)^2}{4} = \frac{16 - 4\pi}{4} = 4 - \pi$

19. Factor

a. $9x^5 - 6x^3$
$3x^3(3x^2 - 2)$

b. $x^2 - 25$
$(x + 5)(x - 5)$

c. $x^2 - 10x + 16$
$(x - 2)(x - 8)$

d. $2x^2 - 16x + 32$
$2(x^2 - 8x + 16)$
$2(x - 4)^2$

e. $3x^2 + 13x - 30$
$(3x - 5)(x + 6)$

20. $f(x) = 5x - 3$

a. Slope?
5

b. y-intercept?
-3

c. x-intercept?
$0 = 5x - 3$
$5x = 3$
$x = \frac{3}{5}$

d. Find the equation of a line that is parallel to this line and has an x-intercept 2 in the form $y = mx + b$.
$m = 5$. Point $(2, 0)$
$y - 0 = 5(x - 2)$
$y = 5x - 10$

e. Find the equation of a line that is perpendicular to the line $y = 5x - 3$ and goes through the point $(1, -2)$ in the form $y = mx + b$.
$m = 5$. $m_\perp = -\frac{1}{5}$. Point $(1, -2)$.
$y + 2 = -\frac{1}{5}(x - 1)$
$y = -\frac{1}{5}x + \frac{1}{5} - \frac{10}{5} = -\frac{1}{5}x - \frac{9}{5}$

f. Create a table of values for $y = 5x - 3$ using $x = 0, 1$ and 2.

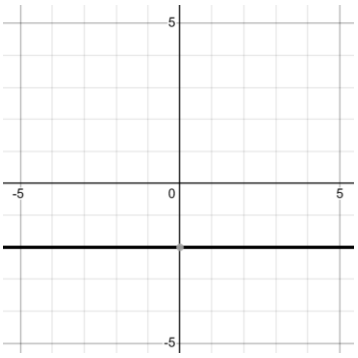
x	$y = 5x - 3$
0	-3
1	2
2	7

g. Evaluate $f(5)$.
$f(5) = 5(5) - 3 = 22$

21. $y = -2$

a. Slope?
0

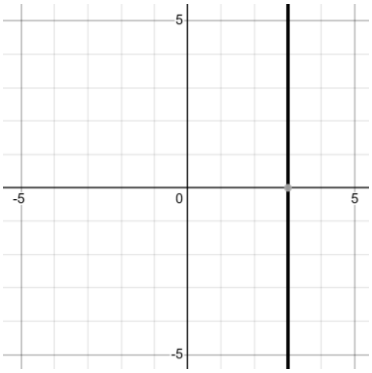
b. Sketch?



22. $x = 3$

a. Slope?
Undefined

b. Sketch



23. Points $A(2, 5)$ and $B(5, 2)$

a. Slope?
$m = \frac{2-5}{5-2} = \frac{-3}{3} = -1$

b. Find the equation in slope-point form: $y - y_1 = m(x - x_1)$ using point A.
$y - 5 = -1(x - 2)$

c. Find the equation in slope-intercept form: $y = mx + b$.
$y = -x + 7$

24. Consider the sequence $7, 9, 11, \ldots$

a. This arithmetic sequence can be written as a line equation $y = 2x + k$. Find k.
$y = 2x + 5$

b. Find the common difference d.
2

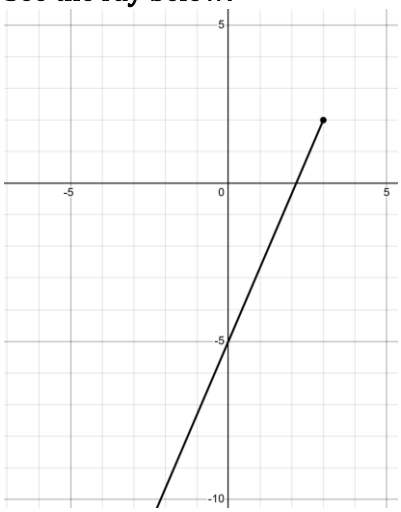
c. Find the 1000^{th} number using the arithmetic formula: $t_n = t_1 + (n - 1)d$.
$t_{1000} = 7 + (1000 - 1)(2) = 2005$

25. $y = 4x - 1$

a. Domain?
$x \in \mathbb{R}$

b. Range?
$y \in \mathbb{R}$

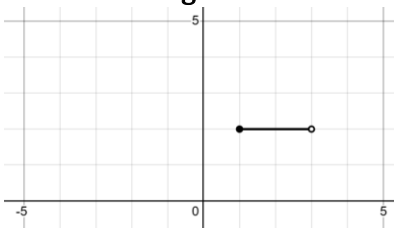
26. See the ray below:



a. Domain?
$x \leq 3$ or $(-\infty, 3]$

b. Range?
$y \leq 2$ or $(-\infty, 2]$

27. See the line segment below:



a. Domain?
$1 \leq x < 3$ or $[1, 3)$

b. Range?
$y = 2$

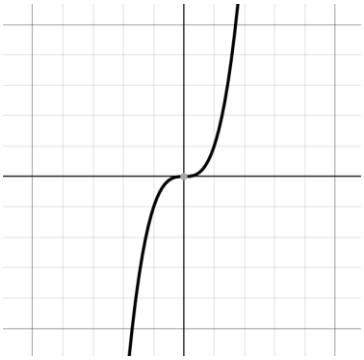
28. Money, M, is a function of time, t, in hours: $M(t) = -10t + 200$.

a. How much money do you initially have?
$M(0) = -10(0) + 200 = 200$

b. When do you run out of money?
$0 = -10t + 200$
$10t = 200$
$t = 20$

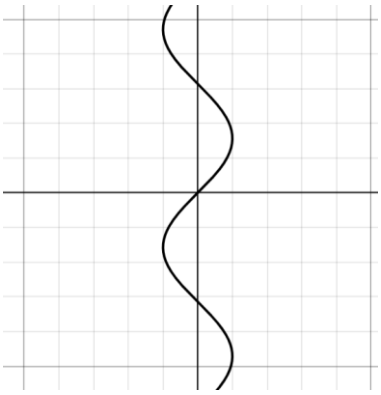
29. Are the following graphs functions?

a. See below:



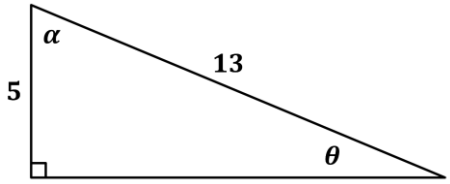
Yes

b. See below:



No

30. See triangle below:



a. Find $\cos\theta$.
$\cos\theta = \frac{12}{13}$

b. Find $\sin\theta$.
$\sin\theta = \frac{5}{13}$

c. Find $\tan\alpha$.
$\tan\alpha = \frac{12}{5}$

d. $\alpha = \cos^{-1}\left(\frac{5}{13}\right)$.

31. Write $\frac{2}{3}y - \frac{3}{4}x = 5$ in general form $Ax + By + C = 0$ where $A > 0$ and A, B, C are integers.
Multiply both sides by 12
$8y - 9x = 60$
$0 = 9x - 8y + 60$
$9x - 8y + 60 = 0$

32. Find the (x, y) coordinates of the point of intersection:
$2x - 1 = 3y$ and $3x + 2y = 5$

$2x - 3y = 1$ [1]
$3x + 2y = 5$ [2]
Multiply [1] × 2 and [2] × 3

$4x - 6y = 2$
$9x + 6y = 15$
Add equations:
$13x = 17$
$x = \frac{17}{13}$

Chapter 14: Final Math 10 Review

1. Find the GCF and LCM of $10, 16, 48$.

2. Find the GCF and LCM of $8x^3y^5, 12x^2y^7, 20x^4y^4$.

3. Prime the prime factorization of 22932.

4. $\sqrt[3]{-64} - 4^2$

5. $-3(-3)^{-3}$

6. $2 + \frac{8}{3} \div \frac{4}{5} - 1$

7. Solve $\frac{2x}{3} + 2 = 3 - x + \frac{x+2}{2}$.

8. Solve $\frac{3}{1-2x} = \frac{2}{3x-2}$.

9. Expand $2(1 - 4x)^2$.

10. Expand $(x - 1)^3$.

11. The diameter of a circle is $4x - 8$. Find the area in the form $ax^2 + bx + c$.

12. A cylinder has a height of $x + 2$ and a radius of x.
Find the lateral (side) area only in the form $ax^2 + bx$.

13. If possible factor:
    a. $8a^2 - 18b^4c^{10}$

    b. $x^2 + 25$

    c. $3x^2 - 12x + 12$

    d. $5x^2 + 32x - 21$

e. $6x^3 + 3x^2 + 8x + 4$

f. $\tan^2\theta - 14\tan\theta + 45$

14. $x^2 + kx - 4$. Find the possible values of k such that this trinomial can be factored.

15. $\left(\frac{2x}{3x^{-2}}\right)^{-3} \div \frac{1}{x}$

16. Solve $27^{2-3x} = 9^{5x-1}$.

17. $A(2, -3)$ and $B(-1, 6)$. Find the equation in the form $y = mx + b$.

18. $A(2, k)$ and $B(5, -4)$. Given the slope of AB is -2, find k.

19. Sketch the line $y = 3 - \frac{2x}{3}$.

20. Sketch $x = -\frac{3}{2}$.

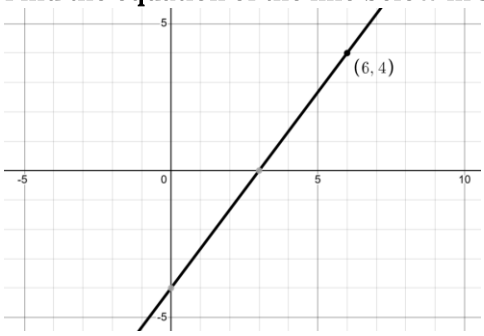
21. Is the point $(-7, 6)$ on the line $2x + 3y = 4$?

22. Given $f(x) = 5x - 3$, evaluate $f(-2)$.

23. Write $2y = -1.\overline{3}x + \frac{3}{2}$ in general form: $Ax + By + C = 0$ where $A > 0$, and variables A, B, C are integers.

24. Given the slope is -3 and the x-intercept is 4, find the equation of the line in the form $y = mx + b$.

25. Find the equation of the line below in slope-point form using the point $(6, 4)$.



26. Given the points $(3, 4)$ and $(5, -7)$
    a. Find the y-intercept.
    b. Find the x-intercept.

27. $10, 13, 16, \ldots$ Find the 200^{th} number.

28. Find the missing number in the table below:

x	y
-5	2
1	8
3	?

29. $L_1 = 3x + 5$. Find the equation of a line that is perpendicular to L_1 and goes through the point $(5, -3)$.

30. Money is a function of time: $M(t) = 80t + 100$. How many hours do you need to work to save $660?

31. For the first three hours you scooter at a rate of 15 kph. For the next two hours you scooter at a rate of 5 kph. Model this on a distance-time graph.

32. Given the equation $Ax + By + C = 0$, what conditions must be true for the graph of the line to have a positive slope and a negative y-intercept?

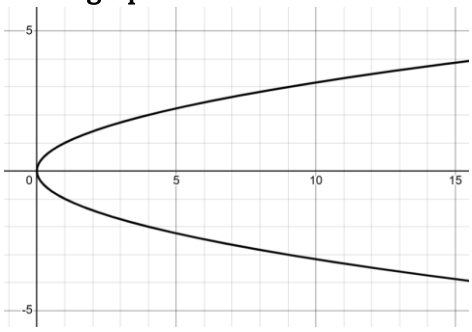
33. Two isosceles triangles have the same height. The slopes of the sides of triangle A are triple the slopes of the corresponding sides of triangle B. How do the lengths of their bases compare?

34. Sketch $f(x) = 3x - 4, x > 2$.

35. Sketch $y = 5, 2 < x \leq 4$.

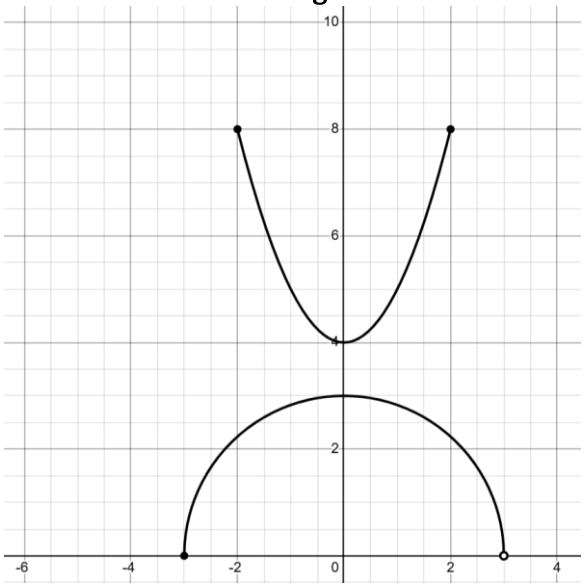
36. Write $x \leq 3$ in set notation using either rounded or square brackets.

37. Is the graph below a function?

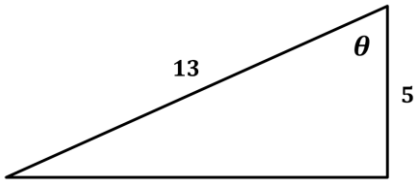


38. Given $f(x) = 2x^2 + 4x$, evaluate $f(3)$.

39. Find the domain and range of the function below:



40. See the right triangle below:



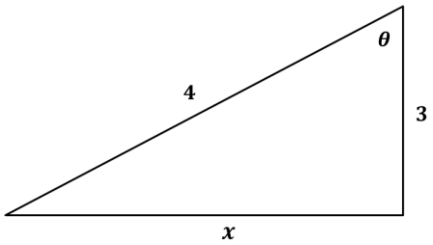
a. Find $\cos\theta$.

b. Find $\tan\theta$.

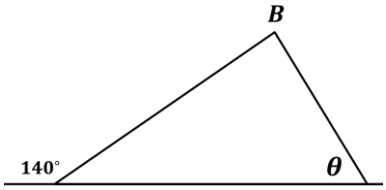
c. $\theta = cos^{-1}(?)$.

d. Given $\alpha = \sin^{-1}\left(\frac{5}{13}\right)$, label α on the triangle above.

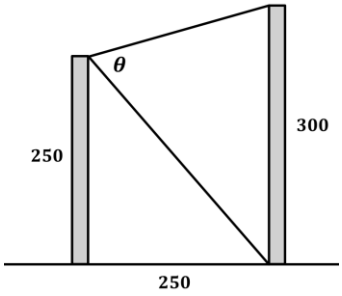
41. Find $\sin\theta$ in the right triangle below:



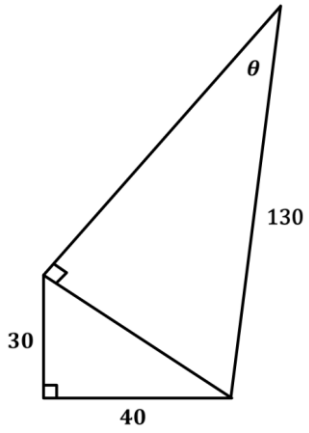
42. Given $\angle B$ in the triangle below is $90°$, find θ.



43. Find θ in the diagram below:



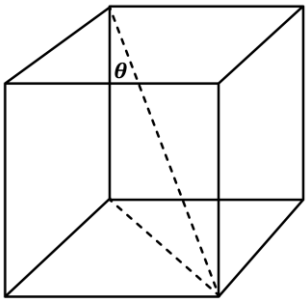
44. See diagram below:



a. Find $\cos\theta$.

b. $\theta = \tan^{-1}(?)$.

45. Find θ in the cube below:



46. $11, 8, 5, \ldots$

a. Find the 100^{th} number.

b. Find S_{100}.

47. The third term is 21 and the 10^{th} term is 63. Find t_5.

48. $4, \frac{9}{2}, 5, \ldots, \frac{25}{2}, 13$. Find the sum of this series.

49. Find the sum of the multiples of 7 between 50 and 200.

50. $S = \sum_0^3 (x^3 - x)$

a. How many terms exist?

b. Evaluate S.

51. Given the system: $x + y = 3$ and $y = 2x - 3$ is $(2, 3)$ a point of intersection?

52. Find the (x, y) coordinates of the point of intersection:

$2x + 3y = 4$

$3y - 4x - 4 = 0$

53. Solve $4x - 3y = 1$ and $2x = \frac{1}{2} + 1.5y$

54. 3 equal sized apples and 6 equal sized bananas weigh 30 lbs.
    2 equal sized apples and 8 equal sized bananas weight 28.
    How much does one banana weigh?

55. A jet flied 600 miles from City A to City B in 2 hours with a tailwind. One the return trip, it takes 2.5 hours against the same wind. If the jet's airspeed (speed without wind) is kph and the wind speed is y kph, set up a system of equations to find x and y.

56. A bartender needs to create 10 liters of a 40% alcohol solution by mixing a 20% alcohol solution with a 60% alcohol solution. Let x represent the liters of the 20% solution and y represent the liters of the 60% solution. Set up a system of linear equations to determine x and y.

57. Suppose your after-tax (net) income is $50,000. If your average tax rate is 20%, how much was your gross salary (before taxes)?