PC11 Types of Numbers Lesson

1. Provide the mathematical symbol and examples of the following types of numbers:
   a. Natural
      $x \in \mathbb{N}$
      ex. 1, 2, 3, 4 (positive whole numbers)

   b. Whole
      $x \in \mathbb{W}$
      ex. 0, 1, 2, 3, 4 (includes 0)

   c. Integers
      $x \in \mathbb{Z}$
      ex. $\ldots -3, -2, -1, 0, 1, 2, 3, \ldots$
      $x \in \mathbb{Z}^-$ refers to negative integers only

   d. Rational
      $x \in \mathbb{Q}$
      An number that can be expressed in the form $\frac{a}{b}$ where $a, b \in \mathbb{Z}$
      ex. $\frac{3}{4}, 2, -0.2, \sqrt{9}$
      Is $2.\overline{6}$ rational?
      $2 + \frac{2}{3} = 2\frac{2}{3} = \frac{8}{3}$

   e. Real
      $x \in \mathbb{R}$
      Includes all types of numbers (except ones that are undefined)
      ex. $5, \sqrt{3}, \pi$

   f. Enrichment: What is a complex number??
      $a + bi$ (a number that consists of a real number plus an imaginary number)
      i is defined to be $\sqrt{-1}$
      $i = \sqrt{-1}$
      $i^2 = -1$

2. Rational or irrational?
   a. $\sqrt{5}$
      Irrational

   b. $\sqrt{16}$
      $= 4 = \frac{4}{1}$
      Rational

c. $\sqrt{\frac{25}{9}}$

$\frac{\sqrt{25}}{\sqrt{9}}$

$= \frac{5}{3}$

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

$\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

Rational

d. π

$3.1415926535 \ldots$

Irrational

e. 0

$= \frac{0}{2}$

Rational

f. $1.\overline{6}$

$1 + \frac{2}{3} = \frac{5}{3}$

Rational

g. 0.02

$\frac{0.02}{1}$

Multiply the numerator and denominator by 100

$= \frac{2}{100}$

Rational

h. 2.5

$\frac{2.5}{1} = \frac{25}{10}$

Rational

i. $\sqrt{1.21}$

$\frac{1.1}{1} = \frac{11}{10}$

Rational

j. $e \approx 2.718$

Irrational

3. Rank from least to greatest: $2.5, \sqrt{9}, -100, \frac{8}{3}, 3.\overline{3}, \infty, 200\%$

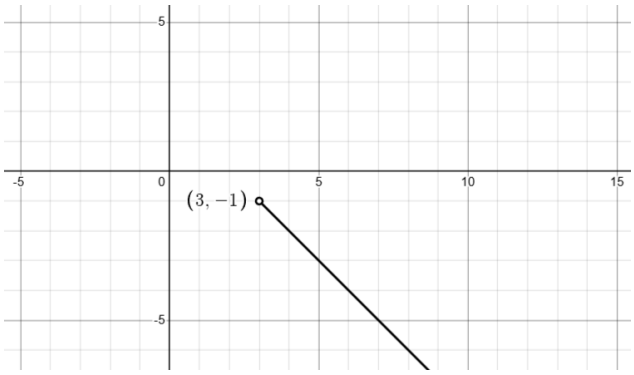
$2.5, 3, -100, 2.\overline{6}, 3.\overline{3}, \infty, 2$

$-100, 200\%, 2.5, \frac{8}{3}, \sqrt{9}, 3.\overline{3}, \infty$

4. Sketch $y = 2 - x, x > 3$

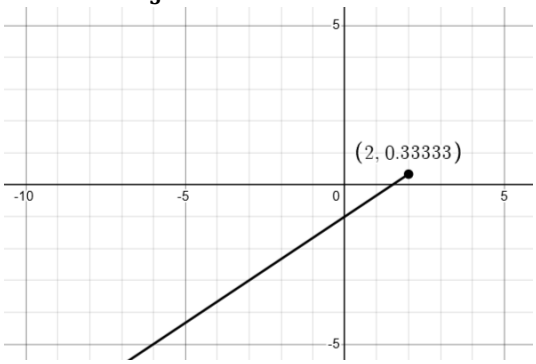
Same as $y = -x + 2, x > 3$

When $x = 3, y = 2 - 3 = -1$



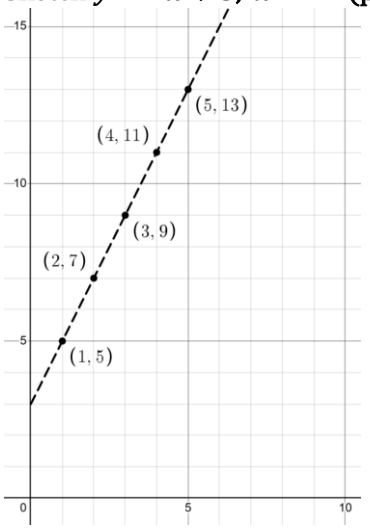
Notice the hollow hole on the endpoint.

5. Sketch $y = \frac{2}{3}x - 1, x \leq 2$



Notice the solid point at the endpoint.

6. Sketch $y = 2x + 3,\ x \in \mathbb{Z}^+$ (positive integer)



Technically we should not include the dashed line.

7. List the four prime numbers
   2, 3, 5, 7
   The number "1" is not considered to be prime numbers and prime numbers must be positive.

8. List the first four positive perfect squares
   1, 4, 9, 16

9. List the first four positive perfect cubes
   1, 8, 27, 64

10. Show that $0.\overline{7}$ is rational

$x = 0.\overline{7} = 0.77777777777777777\ldots$ [1]

$10x = 7.77777777777777777\ldots$ [2]

Subtracting $[2] - [1]$:

$9x = 7$

$x = \frac{7}{9}$

11. 1, 4, 9, 16, 25 ...

What is the value of the n^{th} term?

n^2

12. $0.\overline{k} = \frac{a}{b}$. Find a and b

$\frac{k}{9} \rightarrow a = k$ and $b = 9$

13. Show that $1.\overline{23}$ is a rational number

The answer is $1 + 0.\overline{23}$

Let $x = 0.\overline{23}$

$100x = 23.232323\ldots$

$x = 0.2323232323\ldots$

Subtracting $99x = 23$

$x = \frac{23}{99}$

Thus $1.\overline{23} = 1 + \frac{23}{99} = 1\frac{23}{99} = \frac{99}{99} + \frac{23}{99} = \frac{122}{99}$

14. Show that $2.0\overline{51}$ is a rational number

Focus on $0.0\overline{51}$ part

$x = 0.05151515151\ldots$

$1000x = 51.515151\ldots$

$10x = \quad 0.515151\ldots$

Subtracting $990x = 51$

$x = \frac{51}{990}$

Thus $2 + x = \frac{677}{330}$ which is in the form of a rational number