PC11 Factoring Solutions (DO NOT WRITE ON THIS PAPER)

1. Factor $2x^2 - 4x$
   $2x(x - 2)$

2. Factor by pulling out the GCF:
   $6x^4y^4z - 9x^3y^6z^2 + 3x^3y^4z^2$
   $3x^3y^4z(2x - 3y^2z + z)$

3. True or False:
   a. $(x + k)^2 = x^2 + k^2$
      False

   b. $a^2 + b^2 = (a + b)(a - b)$
      False
      $a^2 - b^2 = (a + b)(a - b)$

   c. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
      True

4. Factor $a^2 - 9$
   $(a + 3)(a - 3)$

5. Factor $9x^2 - 25y^6$
   $(3x^2 + 5y^3)(3x^2 - 5y^3)$

6. Factor $2x^3 - 18x$
   $2x(x^2 - 9)$
   $2x(x + 3)(x - 3)$

7. Factor $x^2 - 3x - 10$
   $(x - 5)(x + 2)$

8. Factor $6x^2 - x - 2$
   $(3x - 2)(2x + 1)$

9. Factor $-2x^2 + 6x + 20$
   $-2(x^2 - 3x - 10)$
   $-2(x - 5)(x + 2)$

10. Factor $200x^2 + 500x - 1200$
    $100(2x^2 + 5x - 12)$
    $100(2x - 3)(x + 4)$

11. Factor $10x^2 + 29x - 21$
    $(5x - 3)(2x + 7)$

12. Factor $x^2(x - 1) + 4(x - 1)$
    $(x - 1)[x^2 + 4]$

13. Factor $x^2(x - 2) + (2 - x)9$
    $x^2(x - 2) - 9(x - 2)$
    $(x - 2)[x^2 - 9]$
    $(x - 2)[x + 3][x - 3]$

14. Factor $(x^2 - 1)^2 - 7(x^2 - 1) + 12$
    Let $a = x^2 - 1$
    $a^2 - 7a + 12$
    $[a - 3][a - 4]$
    $[(x^2 - 1) - 3][(x^2 - 1) - 4]$
    $[x^2 - 4][x^2 - 5]$
    $[x + 2][x - 2][x^2 - 5]$

15. Enrichment: Factor $2(\sin\theta)^2 - 5\sin\theta - 3$
    $2a^2 - 5a - 3$
    $(2a + 1)(a - 3)$
    $(2\sin\theta\ )(\sin\theta - 3)$

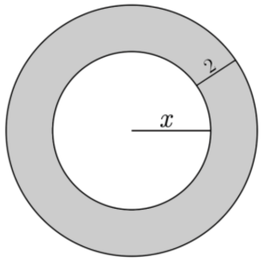
16. Challenge:
    a. Factor $\frac{x^2}{2} + x - 4$
       $= \frac{1}{2}(x^2 + 2x - 8)$
       $= \frac{1}{2}(x + 4)(x - 2)$

    b. Factor $a^2 + 1.5a - 10$
       $\frac{1}{2}(2a^2 + 3a - 20)$
       $\frac{1}{2}(2a - 5)(a + 4)$

    c. Factor $e^{2x} - 25$ $(e \approx 2.72)$
       $(e^x)^2 - (5)^2$
       Let $a = e^x$ and $b = 5$
       $a^2 - b^2$
       $(a + b)(a - b)$
       $(e^x + 5)(e^x - 5)$

    d. Factor by grouping terms: $7a + 7a^3 + a^4 + a^6$
       $7a(1 + a^2) + a^4(1 + a^2)$
       $(1 + a^2)[7a + a^4]$

e. What is the area of the shaded region below in fully factored form?



$A_{\text{shaded}} = A_{\text{big}} - A_{\text{small}}$
$A = \pi R^2 - \pi r^2$
$A = \pi(x+2)^2 - \pi(x)^2$
$A = \pi\left[(x+2)^2 - x^2\right]$
$A = \pi[(x+2)+x][(x+2)-x]$
$A = \pi[2x+2][2]$
$A = \pi 2(x+1)(2)$
$A = 4\pi(x+1)$
You try it by expanding: $A = \pi(x+2)^2 - \pi(x)^2$
$A = \pi\left(x^2+4x+4\right) - \pi x^2$
$A = \pi x^2 + 4\pi x + 4\pi - \pi x^2$
$A = 4\pi x + 4\pi$
$A = 4\pi(x+1)$

f. Find the possible values of k such that
$2x^2 + kx + 8$ can be factored
$(2x+1)(x+8) = 2x^2 + 17x + 8\ (k = 17)$
$(2x+8)(x+1) = 2x^2 + 10x + 8\ (k = 10)$
$(2x+2)(x+4) = 2x^2 + 10x + 8$ (same)
$(2x+4)(x+2) = 2x^2 + 8x + 8\ (k = 8)$
$(2x-1)(x-8) = 2x^2 - 17x + 8\ (k = -17)$
$(2x-8)(x-1) = 2x^2 - 10x + 8\ (k = -10)$
$(2x-2)(x-4) = 2x^2 - 10x + 8$ (same)
$(2x-4)(x-2) = 2x^2 - 8x + 8\ (k = -8)$
6 unique k values

g. Factor $x^2 - y^2 + 2y - 1$
Through trial and error
$(x - y + 1)(x + y - 1)$