PC11 Radicals Solutions (DO NOT WRITE ON THIS PAPER)

1. Evaluate $\sqrt{25}$
   5

2. Solve $x^2 = 25$
   $x = \pm 5$

3. Write as a mixed radical:
   a. $\sqrt{8}$
      $2\sqrt{2}$

   b. $\sqrt{1575}$
      $\sqrt{5 \times 5 \times 3 \times 3 \times 7}$
      $15\sqrt{7}$

4. Write as an entire radical
   a. $2\sqrt{3}$
      $\sqrt{3 \times 2 \times 2}$
      $\sqrt{12}$

   b. $3\sqrt[3]{2}$
      $\sqrt[3]{2 \times 3 \times 3 \times 3}$
      $\sqrt[3]{54}$

   c. $-2\sqrt[3]{3}$
      $-\sqrt[3]{24}$ or $\sqrt[3]{3(-2)(-2)(-2)} = \sqrt[3]{-24}$

5. If possible, evaluate
   a. $\sqrt{-9}$
      No solution

   b. $\sqrt[3]{-8}$
      -2

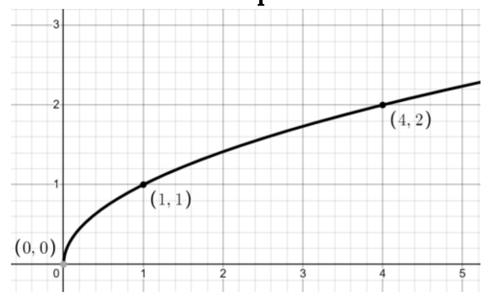
   c. $\sqrt{4\,000\,000}$
      2000

   d. $\sqrt{0.25}$
      0.5

   e. $\sqrt{\frac{4}{9}}$
      $\frac{\sqrt{4}}{\sqrt{9}}$
      $\frac{2}{3}$

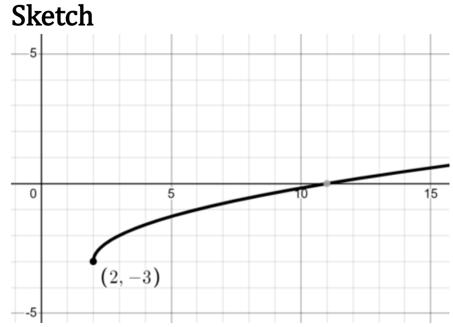
6. Order from least to greatest: $\sqrt{9}, 2\sqrt{3}, \sqrt{30}, \pi$
   $\sqrt{9}, \sqrt{12}, \sqrt{30}, \sqrt{\pi^2}$
   $\sqrt{9}, \pi, \sqrt{12}, \sqrt{30}$

7. $f(x) = \sqrt{x}$
   a. Sketch and label 3 points



   b. Evaluate $f(25)$
      $f(25) = \sqrt{25} = 5$
      i.e. $(25, 5)$ is a point on the radical graph

8. $y = \sqrt{x-2} - 3$
   a. Sketch

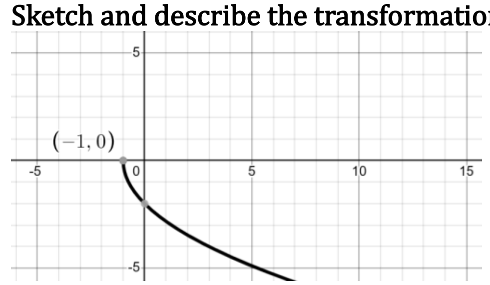


   b. Domain?
      $x \geq 2$ or $[2, \infty)$

   c. Range?
      $y \geq -3$ or $[-3, \infty)$

9. $y = \sqrt{x-a} + b$. Given $a, b > 0$,
   describe the transformation.
   Shift a units left and b units up

10. $y = -2\sqrt{x+1}$
    a. Sketch and describe the transformation



Multiply y's by a factor of -2
Then shift unit left

b. Domain?
$x+1 \geq 0$
$x \geq -1$

c. Range?
$y \leq 0$

11. $y = a\sqrt{x+b}+c$
Given $a, b, c > 0$ describe the transformation.
$\times$ y's by a
b units left or right
c units up or down

12. Find the domain of:

a. $\sqrt{3-5x}$
$3-5x \geq 0$
$3 \geq 5x$
$\frac{3}{5} \geq x$
$x \leq \frac{3}{5}$

b. $\frac{\sqrt{1-2x}}{x}$
$x \neq 0$
$1-2x \geq 0$
$1 \geq 2x$
$\frac{1}{2} \geq x$
$x \leq \frac{1}{2}$

c. $\frac{\sqrt{3x-2}}{x^2-9}$
$x^2-9 \neq 0$
$x^2 \neq 9$
$x \neq \pm 3$

$3x-2 \geq 0$
$3x \geq 2$
$x \geq \frac{2}{3}$

d. $\frac{2\sqrt{x}}{x^2+x-20}$
$\frac{2\sqrt{x}}{(x+5)(x-4)}$
$x \neq -5, 4$
$x \geq 0$ or
$x \neq 4, x \geq 0$

e. $\frac{\sqrt{2+5x}}{3x^2+13x-10}$
$(3x-2)(x+5)$
$x \neq -5$
$3x-2 \neq 0$
$x \neq \frac{2}{3}$
$2+5x \geq 0$
$5x \geq -2$
$x \geq -\frac{2}{5}$

13. Rationalize:

a. $\frac{1}{\sqrt{2}}$
$\frac{\sqrt{2}}{2}$

b. $\frac{4}{\sqrt{8}}$
$\frac{4}{2\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

c. $\frac{9}{6-\sqrt{3}}$
$\frac{9}{6-\sqrt{3}} \cdot \frac{(6+\sqrt{3})}{(6+\sqrt{3})}$
$\frac{9(6+\sqrt{3})}{36-3} = \frac{3}{11}(6+\sqrt{3})$

d. $\frac{5}{5+\sqrt{5}}$
$\frac{5}{5+\sqrt{5}} \cdot \frac{5-\sqrt{5}}{5-\sqrt{5}}$
$\frac{5(5-\sqrt{5})}{25-5} = \frac{5}{20}(5-\sqrt{5}) = \frac{1}{4}(5-\sqrt{5})$

e. $\frac{1}{\sqrt[3]{3}} = \frac{1}{\sqrt[3]{3}} \times \left(\frac{\sqrt[3]{3}}{\sqrt[3]{3}} \cdot \frac{\sqrt[3]{3}}{\sqrt[3]{3}}\right) = \frac{\sqrt[3]{9}}{3}$

14. Simplify $\sqrt{8}+3\sqrt{2}$
$2\sqrt{2}+3\sqrt{2} = 5\sqrt{2}$

15. Simplify $\sqrt{8}-\sqrt[3]{32}+3\sqrt{2}+\sqrt[3]{4}$
$2\sqrt{2}-\sqrt[3]{2\cdot 2\cdot 2\cdot 2\cdot 2}+3\sqrt{2}+\sqrt[3]{4}$
$5\sqrt{2}-2\sqrt[3]{4}+\sqrt[3]{4}$
$5\sqrt{2}-\sqrt[3]{4}$

16. Simplify $\frac{\sqrt{12}}{2}$
$\frac{\sqrt{2\times 2\times 3}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

17. Simplify $\frac{-2+\sqrt{12}}{-2}$
$\frac{-2}{-2}+\frac{\sqrt{2\times 2\times 3}}{-2} = 1-2\sqrt{3}$

18. $2\sqrt{3} \times 3\sqrt{2}$
$6\sqrt{6}$

19. $(5\sqrt{5})(2\sqrt{5})$
$10(5) = 50$

20. $\sqrt{2} \cdot \sqrt{3} \cdot \sqrt{5}$
$\sqrt{30}$

21. $a^b \cdot \sqrt{d} \cdot a^c \cdot \sqrt{e}$
$a^{b+c}\sqrt{de}$

22. Expand and simplify:
   a. $2\sqrt{2}(\sqrt{4} - 3\sqrt{2} + 1)$
   $2\sqrt{2}(2 - 3\sqrt{2} + 1)$
   $2\sqrt{2}(3 - 3\sqrt{2})$
   $6\sqrt{2} - 6(2)$
   $6\sqrt{2} - 12$

   b. $(2 - \sqrt{2})^2$
   $4 - 4\sqrt{2} + 2$
   $6 - 4\sqrt{2}$

   c. $(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})$
   $3 - 2 = 1$

   d. $3(\sqrt{8} - \sqrt{2})(1 - \sqrt{8})$
   $3(2\sqrt{2} - 1\sqrt{2})(1 - 2\sqrt{2})$
   $3\sqrt{2}(1 - 2\sqrt{2})$
   $3\sqrt{2} - 6(2)$
   $3\sqrt{2} - 12$

   e. $(\sqrt{8} - 1)^3$
   $(2\sqrt{2} - 1)(2\sqrt{2} - 1)^2$
   $(2\sqrt{2} - 1)[4(2) - 4\sqrt{2} + 1]$
   $(2\sqrt{2} - 1)[8 - 4\sqrt{2} + 1]$
   $(2\sqrt{2} - 1)[9 - 4\sqrt{2}]$
   $18\sqrt{2} - 8(2) - 9 + 4\sqrt{2}$
   $18\sqrt{2} - 16 - 9 + 4\sqrt{2}$
   $22\sqrt{2} - 25$

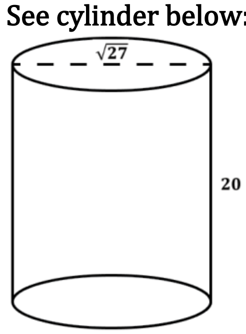
   f. $(\sqrt{2} - \sqrt{3})(2 + \sqrt{6} + 3)$
   $(\sqrt{2} - \sqrt{3})(5 + \sqrt{6})$
   $5\sqrt{2} + \sqrt{12} - 5\sqrt{3} - \sqrt{18}$
   $5\sqrt{2} + 2\sqrt{3} - 5\sqrt{3} - 3\sqrt{2}$
   $2\sqrt{2} - 2\sqrt{3}$

23. A rectangle has a base of $4\sqrt{2} - 2\sqrt{3}$ and a height of $\sqrt{8} - \sqrt{3}$
   a. Area in simplified form?
   $(4\sqrt{2} - 2\sqrt{3})(2\sqrt{2} - \sqrt{3})$
   $8(2) - 4\sqrt{6} - 4\sqrt{6} + 2(3)$
   $22 - 8\sqrt{6}$

   b. Perimeter in simplified form?
   $2(4\sqrt{2} - 2\sqrt{3}) + 2(2\sqrt{2} - \sqrt{3})$
   $8\sqrt{2} - 4\sqrt{3} + 4\sqrt{2} - 2\sqrt{3}$
   $12\sqrt{2} - 6\sqrt{3}$

24. A cylinder has a diameter of $\sqrt{8}$ and a height of 10
   a. Volume?
   $V = \pi r^2 h$
   $d = \sqrt{8} = 2\sqrt{2}$
   $r = \sqrt{2}$
   $V = \pi(\sqrt{2})^2(10)$
   $= 20\pi$
   b. Area including the bottom?
   $A = 2\pi r^2 + \pi d h$
   $2\pi(\sqrt{2})^2 + \pi(2\sqrt{2})(10)$
   $4\pi + 20\sqrt{2}\pi$
   or $\pi(4 + 20\sqrt{2})$

25. See cylinder below:



   a. Volume?
   $d = \sqrt{27} = 3\sqrt{3} \rightarrow r = \frac{3}{2}\sqrt{3}$
   $V = \pi r^2 h$
   $= \pi\left(\frac{3\sqrt{3}}{2}\right)^2(20)$
   $= \pi\left(\frac{27}{4}\right)(20)$
   $= 135\pi$

b. Area including the bottom?

$A = 2\pi r^2 + \pi dh$

$2\pi\left(\frac{3\sqrt{3}}{2}\right)^2 + \pi(3\sqrt{3})(20)$

$2\pi\left(\frac{27}{4}\right) + 60\sqrt{3}\pi$

$\frac{27}{2}\pi + 60\sqrt{3}\pi$

or $\pi\left(60 + \frac{27}{2}\right)$ or 73.5π

26. Solve:

a. $\sqrt{x} = 3$

$x = 9$

b. $2\sqrt{x} = 4$

$\sqrt{x} = 2$

$x = 4$

c. Solve $\sqrt{x-2} = 3$

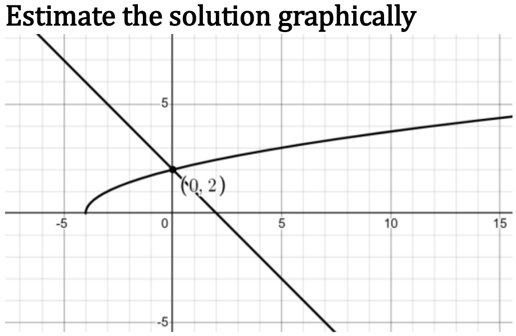
Square both sides

$x - 2 = 9$

$x = 11$

27. $\sqrt{x+4} = 2 - x$

a. Estimate the solution graphically



$x = 0$

b. Find the point of intersection algebraically

$\sqrt{x+4} = 2 - x$

Square both sides

$x + 4 = (2 - x)^2$

$x + 4 = 4 - 4x + x^2$

$0 = x^2 - 5x$

$0 = x(x - 5)$

$x = 0, 5$

But we reject $x = 5$

c. Check for extraneous roots

Since $LS = \sqrt{x+4} = \sqrt{5+4} = 3$

But $RS = 2 - x = 2 - 5 = -3$

$LS \neq RS$ so reject $x = 5$

d. Find the point of intersection

When $x = 0, y = \sqrt{x+4}$

(we can plug in $x = 0$ into either equation)

$y = \sqrt{0+4} = \sqrt{4} = 2$

$(0, 2)$ is the point of intersection

28. Solve $\sqrt{x-1} = 2 - \frac{x}{2}$

Square both sides

$x - 1 = \left(2 - \frac{x}{2}\right)^2$

$x - 1 = 4 - 2x + \frac{x^2}{4}$

$0 = \frac{x^2}{4} - 3x + 5$

Multiply by 4

$0 = x^2 - 12x + 20$

$0 = (x - 10)(x - 2)$

$x = 2, 10$

Reject $x = 10$

29. Solve $\sqrt{3x-2} - 1 = 6 - \frac{x}{2}$

$\sqrt{3x-2} = 7 - \frac{x}{2}$

Square both sides

$3x - 2 = \left(7 - \frac{x}{2}\right)^2$

$3x - 2 = 49 - 7x + \frac{x^2}{4}$

$0 = \frac{x^2}{4} - 10x + 51$

Multiply by 4

$0 = x^2 - 40x + 204$

$0 = (x - 6)(x - 34)$

$x = 6, 34$

Reject $x = 34$

Challenge:

30. Enrichment: Solve $\sqrt{x+1} - 2 = \sqrt{x-3}$

Square both sides

$\left(\sqrt{x+1} - 2\right)^2 = x - 3$

$x + 1 - 4\sqrt{x+1} + 4 = x - 3$

$8 = 4\sqrt{x+1}$

Divide by 4

$2 = \sqrt{x+1}$

Square both sides

$4 = x + 1$

$x = 3$

31. Enrichment:

a. Define: $|a|$

$$a = \begin{cases} a & a \geq 0 \\ -a & a < 0 \end{cases}$$

b. Simplify $\sqrt{x^2}$
$|x|$

c. Simplify $\sqrt{x^4}$
x^2

d. Simplify $\sqrt{x^6}$
$|x^3|$

e. Simplify $\sqrt{a^2b^8c^{14}}$
$|a|b^4|c^7|$