PC11 Factoring Lesson Solutions

We continue factoring this year in Pre-Calculus 11 because factoring is an important part of quadratic functions and equations. This year we factor quadratics and next year we factor higher degree polynomials. By factoring, we reveal the roots of a polynomial which are also known as x-intercepts or solutions.

- Greatest common factor of a polynomial
- Trinomials of the form $ax^2 + bx + c$
- Difference of squares of the form $a^2x^2 - b^2y^2$
- May extend to $a\left(f(x)\right)^2 + b\left(f(x)\right) + c$ and $a^2\left(f(x)\right)^2 - b^2\left(f(x)\right)^2$

1. Factor $2x^2 - 4x$
   $2x(x - 2)$

2. Factor by pulling out the GCF:
   $6x^4y^4z - 9x^3y^6z^2 + 3x^3y^4z^2$
   $3x^3y^4z(2x - 3y^2z + z)$

3. True or False:
   a. $(x + k)^2 = x^2 + k^2$
      False

   b. $a^2 + b^2 = (a + b)(a - b)$
      False
      $a^2 - b^2 = (a + b)(a - b)$

   c. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
      True

4. Factor $a^2 - 9$
   $(a + 3)(a - 3)$

5. Factor $9x^2 - 25y^6$
   $(3x^2 + 5y^3)(3x^2 - 5y^3)$

6. Factor $2x^3 - 18x$
   $2x(x^2 - 9)$
   $2x(x + 3)(x - 3)$

7. Factor $x^2 - 3x - 10$
   $(x - 5)(x + 2)$

8. Factor $6x^2 - x - 2$
   $(3x - 2)(2x + 1)$

9. Factor $-2x^2 + 6x + 20$
   $-2(x^2 - 3x - 10)$
   $-2(x - 5)(x + 2)$

10. Factor $200x^2 + 500x - 1200$
$100(2x^2 + 5x - 12)$
$100(2x - 3)(x + 4)$

11. Factor $10x^2 + 29x - 21$
$(5x - 3)(2x + 7)$

12. Factor $x^2(x - 1) + 4(x - 1)$
$(x - 1)[x^2 + 4]$

13. Factor $x^2(x - 2) + (2 - x)9$
$x^2(x - 2) - 9(x - 2)$
$(x - 2)[x^2 - 9]$
$(x - 2)[x + 3][x - 3]$

14. Factor $\left(x^2 - 1\right)^2 - 7\left(x^2 - 1\right) + 12$
Let $a = x^2 - 1$
$a^2 - 7a + 12$
$[a - 3][a - 4]$
$\left[\left(x^2 - 1\right) - 3\right]\left[\left(x^2 - 1\right) - 4\right]$
$\left[x^2 - 4\right]\left[x^2 - 5\right]$
$[x + 2][x - 2]\left[x^2 - 5\right]$

15. Enrichment: Factor $2(\sin\theta)^2 - 5\sin\theta - 3$
$2a^2 - 5a - 3$
$(2a + 1)(a - 3)$
$(2\sin\theta\,)(\sin\theta - 3)$

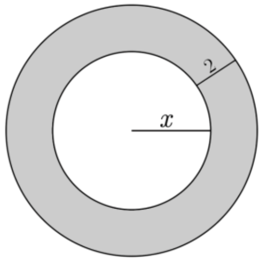
16. Challenge:
a. Factor $\frac{x^2}{2} + x - 4$
$= \frac{1}{2}\left(x^2 + 2x - 8\right)$
$= \frac{1}{2}(x + 4)(x - 2)$

b. Factor $a^2 + 1.5a - 10$
$\frac{1}{2}(2a^2 + 3a - 20)$
$\frac{1}{2}(2a - 5)(a + 4)$

c. Factor $e^{2x} - 25$ ($e \approx 2.72$)
$(e^x)^2 - (5)^2$
Let $a = e^x$ and $b = 5$
$a^2 - b^2$
$(a + b)(a - b)$
$(e^x + 5)(e^x - 5)$

d. Factor by grouping terms: $7a + 7a^3 + a^4 + a^6$
$7a\left(1 + a^2\right) + a^4\left(1 + a^2\right)$
$\left(1 + a^2\right)[7a + a^4]$

e. What is the area of the shaded region below in fully factored form?



$A_{\text{shaded}} = A_{\text{big}} - A_{\text{small}}$
$A = \pi R^2 - \pi r^2$
$A = \pi(x+2)^2 - \pi(x)^2$
$A = \pi\left[(x+2)^2 - x^2\right]$
$A = \pi[(x+2) + x][(x+2) - x]$
$A = \pi[2x+2][2]$
$A = \pi 2(x+1)(2)$
$A = 4\pi(x+1)$
You try it by expanding: $A = \pi(x+2)^2 - \pi(x)^2$
$A = \pi\left(x^2 + 4x + 4\right) - \pi x^2$
$A = \pi x^2 + 4\pi x + 4\pi - \pi x^2$
$A = 4\pi x + 4\pi$
$A = 4\pi(x+1)$

f. Find the possible values of k such that
$2x^2 + kx + 8$ can be factored
$(2x+1)(x+8) = 2x^2 + 17x + 8\ (k = 17)$
$(2x+8)(x+1) = 2x^2 + 10x + 8\ (k = 10)$
$(2x+2)(x+4) = 2x^2 + 10x + 8$ (same)
$(2x+4)(x+2) = 2x^2 + 8x + 8\ (k = 8)$

$(2x-1)(x-8) = 2x^2 - 17x + 8\ (k = -17)$
$(2x-8)(x-1) = 2x^2 - 10x + 8\ (k = -10)$
$(2x-2)(x-4) = 2x^2 - 10x + 8$ (same)
$(2x-4)(x-2) = 2x^2 - 8x + 8\ (k = -8)$
6 unique k values

g. Factor $x^2 - y^2 + 2y - 1$
Through trial and error
$(x - y + 1)(x + y - 1)$