Rational Functions and Equations

This year, in Pre-Calculus 11, we focus on simplifying rational expressions and introduce a few word problems. Next year you will better understand how to graph rational functions.

- Simplifying and applying operations to rational expressions
- Identifying non-permissible values
- Solving equations and identifying and extraneous roots

1. What is a rational expression?

2. Is $f(x) = x^3 + 2x^2 - 3 + \frac{3}{x}$ a polynomial function?

3. Is $h(t) = \sqrt{2}t^5 - \frac{\pi t^3}{2.1} + e^{\pi}$ a polynomial function?

4. Simplify using trinomial factoring:
   $\frac{3x-6}{2x^2+x-10}$

5. Simplify by using difference of squares: $\frac{1-x}{x^2-1}$

6. $f(x) = \frac{1}{x^2-9}$.
   a. What are the non-permissible values?

   b. Domain?

7. $f(x) = \frac{x}{1-3x}$
   a. Solve

   b. Domain?

8. $f(x) = \frac{(2x+1)(x-2)}{x^2+1}$
   a. Solve

   b. Domain?

9. Multiplying rational expressions - Simplify and find the domain of: $\frac{x^2-x-12}{x^2-9} \times \frac{x^2-4x+3}{x^2-4x}$

10. Dividing rational expression - Simplify and find the domain of: $\frac{x^2-4}{x^2-2x} \div \frac{x^2+3x+2}{x}$

11. Simplify: $\frac{2x^2-7x-15}{2x^2-10x} \div \frac{4x^2-9}{6} \times (3-2x)$

12. $\frac{8a^2-2a-3}{a^2-1} \div \frac{2a^2-3a-2}{2a-2} \div \frac{3-4a}{a+1}$
   a. Simplify this rational expression

   b. What are the non-permissible values?

13. The area of a rectangle is $x^2 - 9$. The length of one side is $\frac{x^2-2x-3}{x+1}$. Find the length of the other side.

14. Simplify $\frac{2x}{y} - \frac{x-1}{y}$

15. Write as a single term $\frac{2x}{xy} + \frac{4}{x^2}$

16. Adding rational functions – simplify: $\frac{a^2-20}{a^2-4} + \frac{a-2}{a+2}$

17. Subtracting rational functions – simplify:
$\frac{(x-2)^2}{(x-2)(x+2)} - \frac{x^2-4x+4}{4-x^2}$

18. BEDMAS rational functions – simplify:
$\frac{x+1}{x+6} - \frac{x^2-4}{x^2+2x} \div \frac{2x^2+13x+6}{2x^2+x}$

19. Solve $\frac{-x+5}{(x-5)(x+5)} = 5$

20. Solve $\frac{2}{x-2} + \frac{1}{x} = -1$

21. Solve $\frac{2x+1}{x-4} = \frac{x-3}{x+1}$

22. Solve $\frac{4x-1}{x+2} - \frac{x+1}{x-2} = \frac{x^2-4x+24}{x^2-4}$
Check your answer for extraneous roots.

23. Simplify $\frac{1+\frac{1}{x}}{x-\frac{1}{x}}$

24. What is the halfway point between:

a. $2\frac{2}{3}$ and $\frac{17}{4}$?

b. $\frac{3}{a}$ and $\frac{7}{2a}$?

25. An image found by a convex lens is described by the equation: $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$. Find f

26. Simplify:

$$\left(\frac{p}{p-x} + \frac{q}{q-x} + \frac{r}{r-x}\right) - \left(\frac{x}{p-x} + \frac{x}{q-x} + \frac{x}{r-x}\right)$$

27. Rational functions word problems:

a. The sum of a number and twice its reciprocal is $\frac{9}{2}$. Find the number.

b. Find two consecutive even integers whose reciprocals sum to be $\frac{11}{60}$.

c. Tap A fills the tub in 4 hours. Tab B fills the tub in 2 hours. How long does it take to fill the tub when tap A and B work together?

d. You travel 120 km to Whistler by car, and then return by bus. The average speed of the car is 15 km/h greater than the average speed of the bus.
Express the total time of your trip as a single term.

e. On your first six tests you average a score of 36/50. What average mark must you receive on the next four tests so that your average is 80% in the course?