

# Rational Functions and Equations (solutions)

This year, in Pre-Calculus 11, we focus on simplifying rational expressions and introduce a few word problems. Next year you will better understand how to graph rational functions.

- Simplifying and applying operations to rational expressions
- Identifying non-permissible values
- Solving equations and identifying and extraneous roots

1. What is a rational expression?

A rational function is a function of the form:  $f(x) = \frac{P(x)}{Q(x)}$  where  $P(x)$  and  $Q(x)$  are polynomial functions.

2. Is  $f(x) = x^3 + 2x^2 - 3 + \frac{3}{x}$  a polynomial function?

No

3. Is  $h(t) = \sqrt{2}t^5 - \frac{\pi t^3}{2.1} + e^\pi$  a polynomial function?

Yes

4. Simplify using trinomial factoring:

$$\frac{\frac{3x-6}{2x^2+x-10}}{\frac{3(x-2)}{(x-2)(2x+5)}} = \frac{3}{2x+5}$$

5. Simplify by using difference of squares:  $\frac{1-x}{x^2-1}$

$$\frac{-(x-1)}{(x-1)(x+1)} = -\frac{1}{x+1}$$

6.  $f(x) = \frac{1}{x^2-9}$ .

a. What are the non-permissible values?

$$x^2 - 9 = 0$$

$$x^2 = 9$$

$$x = \pm 3$$

b. Domain?

$$x \neq \pm 3$$

7.  $f(x) = \frac{x}{1-3x}$

a. Solve

$$0 = \frac{x}{1-3x}$$

$$0 = x$$

b. Domain?

$$1 - 3x \neq 0$$

$$1 \neq 3x$$

$$x \neq \frac{1}{3}$$

8.  $f(x) = \frac{(2x+1)(x-2)}{x^2+1}$

a. Solve

$$0 = (2x + 1)(x - 2)$$

$$x = -\frac{1}{2}, 2$$

b. Domain?

$$x^2 + 1 \neq 0$$

$$x^2 \neq -1$$

This is impossible thus  $x \in \mathbb{R}$

9. Multiplying rational expressions - Simplify and find the domain of:  $\frac{x^2-x-12}{x^2-9} \times \frac{x^2-4x+3}{x^2-4x}$

$$\frac{(x+3)(x-4)}{(x+3)(x-3)} \times \frac{(x-3)(x-1)}{x(x-4)}$$

$$\frac{1}{(x-3)} \times \frac{(x-3)(x-1)}{x}$$

$$\frac{x-1}{x}$$

$$x \neq \pm 3, 0, 4$$

10. Dividing rational expression - Simplify and find the domain of:  $\frac{x^2-4}{x^2-2x} \div \frac{x^2+3x+2}{x}$

$$\frac{(x+2)(x-2)}{x(x-2)} \times \frac{x}{(x+2)(x+1)}$$

$$(x+2) \times \frac{1}{(x+2)(x+1)}$$

$$\frac{1}{x+1}$$

$$x \neq 0, \pm 2, -1$$

11. Simplify:  $\frac{2x^2-7x-15}{2x^2-10x} \div \frac{4x^2-9}{6} \times (3-2x)$

$$\frac{(2x+3)(x-5)}{2x(x-5)} \times \frac{6}{(2x+3)(2x-3)} \times -(2x-3)$$

$$\frac{2x+3}{2x} \times \frac{6}{2x+3} \times -1$$

$$-\frac{3}{x}$$

12.  $\frac{8a^2-2a-3}{a^2-1} \div \frac{2a^2-3a-2}{2a-2} \div \frac{3-4a}{a+1}$

a. Simplify this rational expression

$$\frac{(2a+1)(4a-3)}{(a+1)(a-1)} \times \frac{2(a-1)}{(2a+1)(a-2)} \times \frac{a+1}{-(4a-3)}$$

$(2a+1)$  cancels

$(4a-3)$  cancels

$(a+1)$  cancels

$(a-1)$  cancels

$$\frac{2}{a-2} \times -1$$

$$-\frac{2}{a-2}$$

b. What are the non-permissible values?

$$a = \pm 1, -\frac{1}{2}, 2, \frac{3}{4}$$

13. The area of a rectangle is  $x^2 - 9$ . The length of one side is  $\frac{x^2-2x-3}{x+1}$ . Find the length of the other side.

$$? \times \frac{x^2-2x-3}{x+1} = x^2 - 9$$

$$? = (x+3)(x-3) \div \frac{x^2-2x-3}{x+1}$$

$$? = (x+3)(x-3) \times \frac{(x+1)}{(x-3)(x+1)}$$

$$? = x + 3$$

14. Simplify  $\frac{2x}{y} - \frac{x-1}{y}$

$$\frac{2x-(x-1)}{y} = \frac{x+1}{y}$$

15. Write as a single term  $\frac{2x}{xy} + \frac{4}{x^2}$

$$\frac{2}{y} + \frac{4}{x^2}$$

$$\frac{2x^2y}{x^2y} + \frac{4y}{x^2y} = \frac{2x^2y+4y}{x^2y}$$

16. Adding rational functions – simplify:  $\frac{a^2-20}{a^2-4} + \frac{a-2}{a+2}$

$$\frac{a^2-20}{(a+2)(a-2)} + \frac{(a-2)}{(a+2)}$$

$$\frac{a^2-20}{(a+2)(a-2)} + \frac{(a-2)(a-2)}{(a+2)(a-2)}$$

$$\frac{a^2-20+a^2-4a+4}{(a+2)(a-2)}$$

$$\frac{2(a+2)(a-4)}{(a+2)(a-2)}$$

$$\frac{2(a-4)}{a-2}$$

17. Subtracting rational functions – simplify:

$$\frac{(x-2)^2}{(x-2)(x+2)} - \frac{x^2-4x+4}{4-x^2}$$

$$\frac{(x-2)(x-2)}{(x-2)(x+2)} - \frac{(x-2)(x-2)}{-(x^2-4)}$$

$$\frac{(x-2)}{(x+2)} + \frac{(x-2)(x-2)}{(x+2)(x-2)}$$

$$\frac{2(x-2)}{x+2}$$

18. BEDMAS rational functions – simplify:

$$\frac{x+1}{x+6} - \frac{x^2-4}{x^2+2x} \div \frac{2x^2+13x+6}{2x^2+x}$$

$$\frac{x+1}{x+6} - \frac{(x+2)(x-2)}{x(x+2)} \times \frac{x(2x+1)}{(2x+1)(x+6)}$$

$$\frac{x+1}{x+6} - \frac{x-2}{x} \cdot \frac{x}{x+6}$$

$$\frac{x+1}{x+6} - \frac{x-2}{x+6}$$

$$\frac{3}{x+6}$$

19. Solve  $\frac{-x+5}{(x-5)(x+5)} = 5$

$$-x + 5 = 5(x - 5)(x + 5)$$

$$-x + 5 = 5(x^2 - 25)$$

$$-x + 5 = 5x^2 - 125$$

$$0 = 5x^2 + x - 130$$

$$0 = (x - 5)(5x + 26)$$

$$x = 5 \text{ (reject)}, -\frac{26}{5}$$

20. Solve  $\frac{2}{x-2} + \frac{1}{x} = -1$

$$\frac{2x}{(x-2)x} + \frac{1(x-2)}{x(x-2)} = -\frac{x(x-2)}{x(x-2)}$$

**Multiply by the denominator**

$$2x + x - 2 = -x^2 + 2x$$

$$x^2 + x - 2 = 0$$

$$(x + 2)(x - 1) = 0$$

$$x = -2, 1$$

21. Solve  $\frac{2x+1}{x-4} = \frac{x-3}{x+1}$

**Cross multiply**

$$(2x + 1)(x + 1) = (x - 3)(x - 4)$$

$$2x^2 + 3x + 1 = x^2 - 7x + 12$$

$$x^2 + 10x - 11 = 0$$

$$(x + 11)(x - 1) = 0$$

$$x = -11, 1$$

22. Solve  $\frac{4x-1}{x+2} - \frac{x+1}{x-2} = \frac{x^2-4x+24}{x^2-4}$

**Check your answer for extraneous roots.**

$$\frac{(4x-1)(x-2)}{(x+2)(x-2)} - \frac{(x+1)(x+2)}{(x-2)(x+2)} = \frac{x^2-4x+24}{(x+2)(x-2)}$$

**Multiply by the denominator**

$$(4x - 1)(x - 2) - (x + 1)(x + 2) = x^2 - 4x + 24$$

$$4x^2 - 9x + 2 - (x^2 + 3x + 2) = x^2 - 4x + 24$$

$$4x^2 - 9x + 2 - x^2 - 3x - 2 - x^2 + 4x - 24 = 0$$

$$2x^2 - 8x - 24 = 0$$

$$2(x + 2)(x - 6) = 0$$

$$x = -2 \text{ (reject)}, 6$$

23. Simplify  $\frac{1+\frac{1}{x}}{x-\frac{1}{x}}$

$$\left(\frac{x}{x} + \frac{1}{x}\right) \div \left(\frac{x^2}{x} - \frac{1}{x}\right)$$

$$\frac{x+1}{x} \div \frac{x^2-1}{x}$$

$$\frac{x+1}{x} \times \frac{x}{(x+1)(x-1)}$$

$$\frac{1}{x-1}$$

24. What is the halfway point between:

a.  $2\frac{2}{3}$  and  $\frac{17}{4}$ ?

$$\frac{\frac{8}{3} + \frac{17}{4}}{2} = \frac{4}{3} + \frac{17}{8} = \frac{83}{24}$$

b.  $\frac{3}{a}$  and  $\frac{7}{2a}$ ?

$$\frac{\frac{3}{a} + \frac{7}{2a}}{2} = \frac{3}{2a} + \frac{7}{4a} = \frac{6}{4a} + \frac{7}{4a} = \frac{13}{4a}$$

25. An image found by a convex lens is described by the equation:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}. \text{ Find } f$$

$$\frac{1}{f} = \frac{v}{uv} + \frac{u}{uv}$$

$$\frac{1}{f} = \frac{u+v}{uv}$$

$$f = \frac{uv}{u+v}$$

26. Simplify:

$$\left(\frac{p}{p-x} + \frac{q}{q-x} + \frac{r}{r-x}\right) - \left(\frac{x}{p-x} + \frac{x}{q-x} + \frac{x}{r-x}\right)$$

$$\frac{p-x}{p-x} + \frac{q-x}{q-x} + \frac{r-x}{r-x}$$

$$1 + 1 + 1 = 3$$

27. Rational functions word problems:

a. The sum of a number and twice its reciprocal is  $\frac{9}{2}$ . Find the number.

$$x + \frac{2}{x} = \frac{9}{2}$$

Multiply by  $2x$

$$2x^2 + \frac{4x}{x} = \frac{18x}{2}$$

$$2x^2 + 4 = 9x$$

$$2x^2 - 9x + 4 = 0$$

$$(2x - 1)(x - 4) = 0$$

$$x = \frac{1}{2} \text{ or } 4$$

b. Find two consecutive even integers whose reciprocals sum to be  $\frac{11}{60}$ .

$$\frac{1}{x} + \frac{1}{x+2} = \frac{11}{60}$$

$$\frac{1(x+2)}{x(x+2)} + \frac{x}{x(x+2)} = \frac{11x(x+2)}{60x(x+2)}$$

$$\frac{60(x+2)}{60x(x+2)} + \frac{60x}{60x(x+2)} = \frac{11x(x+2)}{60x(x+2)}$$

Multiply by the denominator

$$60x + 120 + 60x = 11x^2 + 22x$$

$$0 = 11x^2 - 98x - 120$$

$$0 = (x - 10)(11x + 12)$$

$$x = 10, 12$$

- c. Tap A fills the tub in 4 hours. Tap B fills the tub in 2 hours.  
How long does it take to fill the tub when tap A and B work together?

$$\frac{t}{4} + \frac{t}{2} = 1$$

Multiply by 4

$$t + 2t = 4$$

$$3t = 4$$

$$t = \frac{4}{3}$$

Tap A fills the tub in 4 hours.

This means that in 1 hour Tap A fills  $\frac{1}{4}$ th of the tub.

Tap B fills the tub in 2 hours.

This means that in 1 hour Tap B fills  $\frac{1}{2}$  of the tub.

If a tap fills the tub in 100 hours,

it means that in 1 hour it fills  $\frac{1}{100}$  th of the tub

Thus if a tap fills the tub in  $t$  hours,

it means that in 1 hour it fills  $\frac{1}{t}$  th of the tub

How much gets filled?

time  $\times$  rate

$$\frac{t}{4} + \frac{t}{2} = 1$$

$$t \times \frac{1}{4} + t \times \frac{1}{2} = t \times \frac{1}{t}$$

$$\frac{t}{4} + \frac{t}{2} = 1$$

- d. You travel 120 km to Whistler by car, and then return by bus.  
The average speed of the car is 15 km/h greater than the average speed of the bus. Express the total time of your trip as a single term.

Let speed of bus be  $x$

Speed of car is  $x + 15$

$$d = vt$$

On the way to Whistler

$$120 = (x + 15)t_1$$

$$t_1 = \frac{120}{x+15}$$

On the way back

$$120 = x t_2$$

$$t_2 = \frac{120}{x}$$

Total time is  $t_1 + t_2$

$$\frac{120}{x+15} + \frac{120}{x}$$

$$\frac{120x}{x(x+15)} + \frac{120(x+15)}{x(x+15)}$$

$$\frac{120x+120x+1800}{x(x+15)}$$
$$\frac{140x+1800}{x(x+15)} \text{ or } \frac{20(7x+90)}{x(x+15)}$$

- e. On your first six tests you average a score of 36/50.  
What average mark must you receive on the next four tests  
so that your average is 80% in the course?

Let  $x$  be your average mark over the next four tests

$$6\left(\frac{36}{50}\right) + 4x = 10 \cdot \frac{80}{100}$$

$$\frac{108}{25} + 4x = \frac{40}{5}$$

Multiply by 25

$$108 + 100x = 200$$

$$100x = 92$$

$$x = 0.92 = 92\%$$