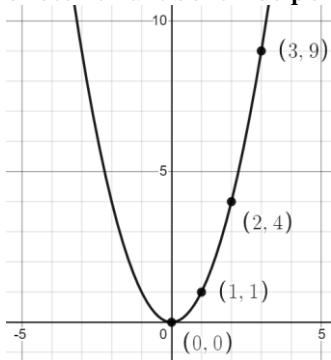


# PC11 Quadratic Assignment Solutions

Name: \_\_\_\_\_

1.  $y = x^2$

a. Sketch and label three points



b. Domain?

$x \in \mathbb{R}$  or  $-\infty < x < \infty$  or  $(-\infty, \infty)$  or "all real numbers"

c. Range?

$y \geq 0$  or  $[0, \infty)$

d. Coordinates of the vertex?

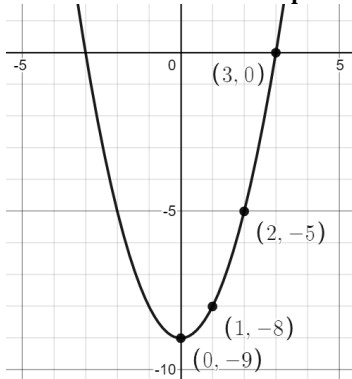
$(0, 0)$

e. Equation of the line of symmetry?

$x = 0$

2.  $f(x) = x^2 - 9$

a. Sketch and label three points



b. Range?

$y \geq -9$  or  $y \in [-9, \infty)$

c. Evaluate  $f(-2)$

$$f(x) = x^2 - 9 = (-2)^2 - 9 = 4 - 9 = -5$$

d. Intercepts?

y-intercept: set  $x = 0$

$$f(0) = (0)^2 - 9 = -9$$

x-intercepts: set  $y = 0$

$$0 = x^2 - 9$$

$$9 = x^2$$

Take the square root both sides

If you have a variable squared (such as  $x$ ) then you have 2 possible answers)

$$\pm\sqrt{9} = x = \pm 3$$

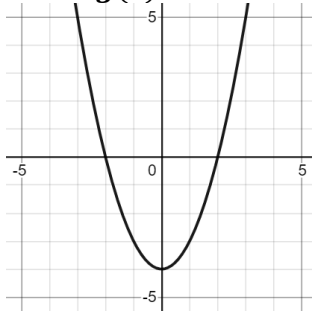
3. Basic Transformations:

$$f(x) = x^2 \text{ and } g(x) = f(x) - 4$$

a. What is the actual equation of  $g(x)$ ?

$$g(x) = (x^2) - 4 = x^2 - 4$$

b. Sketch  $g(x)$



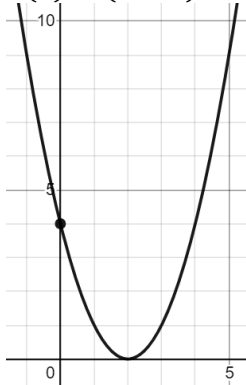
4. Given  $y = f(x) + k$ , what is the transforming effect of  $k$  on the graph  $y = f(x)$ ?

When  $k > 0$  we shift the graph  $f(x)$  up by  $k$  units

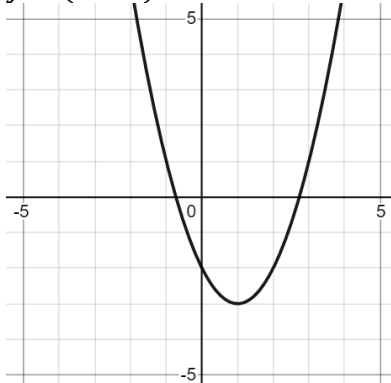
When  $k < 0$  we shift the graph  $f(x)$  down by  $k$  units

5. Sketch the quadratic:

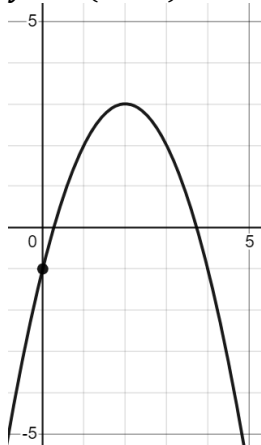
a.  $h(x) = (x - 2)^2$



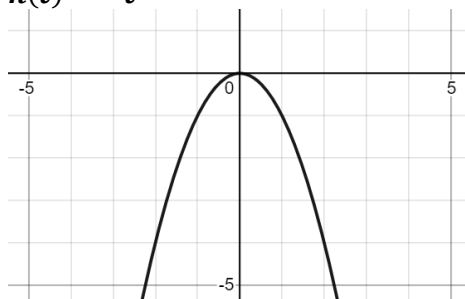
b.  $y = (x + 1)^2 - 3$



c.  $y = -(x - 2)^2 + 3$



d.  $h(t) = -t^2$

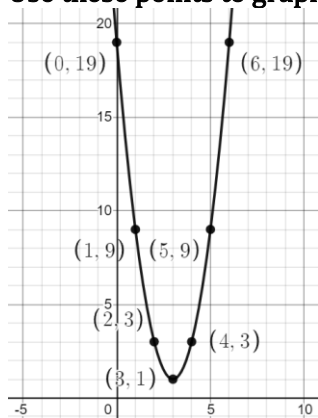


6.  $z(x) = 2(x - 3)^2 + 1$

a. Create a table of values for the function  $z(x)$

0	19
1	9
2	3
3	1
4	3
5	9
6	19

b. Use these points to graph  $z(x)$



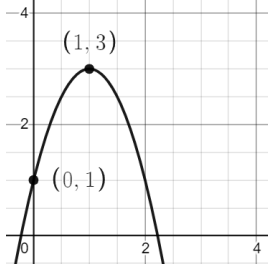
c. Find the z-intercept

Set  $x = 0$

$$z(0) = 2(0 - 3)^2 + 1 = 2(9) + 1 = 19$$

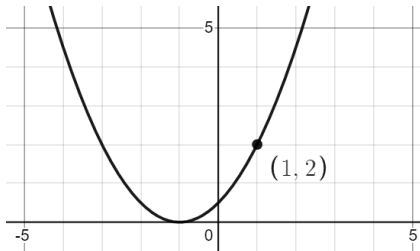
$(0, 19)$  are the coordinates of the z-intercept

7. Sketch  $y = -2(x - 1)^2 + 3$



8.  $f(x) = \frac{1}{2}(x + 1)^2$

a. Sketch  $y = f(x)$



b. Evaluate  $f(1)$   
2

9.  $y = a(x \pm b)^2 \pm c$ . In general, what are the effects of parameters  $a$ ,  $b$ , and  $c$ ?  
Multiply  $y$ 's by  $a$ , shift left or right by  $b$ , shift up or down by  $c$ .
10. Enrichment:  $y = af(b(x \pm c)) \pm d$ . In general, what are the effects of parameters  $a$ ,  $b$ ,  $c$ , and  $d$ ?  
Multiply  $y$ 's by  $a$ , multiply  $x$ 's by  $\frac{1}{b}$ , shift left or right by  $b$ , shift up or down by  $c$ .
11. Enrichment:  $f(x) = x^2$ .  $g(x) = 9f(x)$  and  $h(x) = f(3x)$ . Show that  $g(x) = h(x)$ .  
 $g(x) = 9x^2$ .  $h(x) = (3x)^2 = 9x^2$ . A vertical transformation can also be thought of as a horizontal transformation.

12. Solve a factored Quadratic:

Find the x-intercepts:  $y = (x - 2)(2x + 1)$

Solving an equation means we are finding the x-intercepts or finding the roots.

We set  $y = 0$

$$0 = (x - 2)(2x + 1)$$

$$(x - 2) = 0$$

$$x = 2$$

$$\text{Or } (2x + 1) = 0$$

$$2x = -1$$

$$x = -1/2$$

13. Solve a quadratic that only has one solution:

Find the x-intercept:  $y = 2x^2 - 12x + 18$

$$0 = 2(x^2 - 6x + 9)$$

Divide both sides by 2

$$0 = x^2 - 6x + 9$$

$$0 = (x - 3)(x - 3)$$

$$0 = (x - 3)^2$$

Square root both sides

$$0 = x - 3$$

$$3 = x$$

14. Find the x-intercepts using algebra:  $y = 12 - 3x^2$

$$0 = 12 - 3x^2$$

Divide by 3

$$0 = 4 - x^2$$

$$x^2 = 4$$

Square root both sides

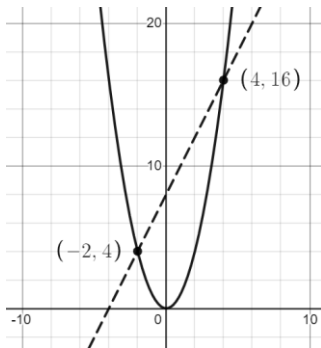
$$x = \pm 2$$

15. Solve  $x^2 = 2x + 8$

a. Graphically by sketching two graphs

$$y_1 = x^2$$

$$y_2 = 2x + 8$$



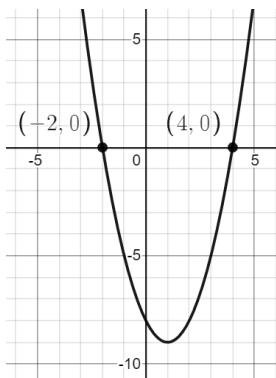
$$x = -2 \text{ or } x = 4$$

b. Graphically by sketching one graph

$$x^2 = 2x + 8$$

$$x^2 - 2x - 8 = 0$$

$$x^2 - 2x - 8 = y$$



$$\text{Intercepts: } x = -2 \text{ or } x = 4$$

c. Algebraically

$$y = x^2 - 2x - 8$$

$$y = (x + 2)(x - 4)$$

$$0 = (x + 2)(x - 4)$$

$$\therefore x = -2 \text{ or } x = 4$$

16. Find the coordinates of the point of intersection:

$$f(x) = (x - 2)^2 \text{ and } g(x) = -(x + 1)^2 + 5$$

$$(x - 2)^2 = -(x + 1)^2 + 5$$

$$(x - 2)(x - 2) = -(x + 1)(x + 1) + 5$$

$$x^2 - 4x + 4 = -(x^2 + 2x + 1) + 5$$

$$x^2 - 4x + 4 = -x^2 - 2x - 1 + 5$$

$$2x^2 - 2x = 0$$

Divide by 2

$$x^2 - x = 0$$

$$x(x - 1) = 0$$

$$x = 0 \text{ or } x = 1$$

$$\text{When } x = 0, y = (x - 2)^2 = 4 \rightarrow (0, 4)$$

$$\text{When } x = 1, y = (x - 2)^2 = 1 \rightarrow (1, 1)$$

17. Complete the square and identify the coordinates of the vertex:

a.  $y = x^2 + 4x + 1$

$$y = (x + 2)^2 - 2^2 + 1$$

$$y = (x + 2)^2 - 4 + 1$$

$$y = (x + 2)^2 - 3$$

$$V(-2, -3)$$

b.  $y = x^2 - 2x$

$$y = (x - 1)^2 - 1^2$$

$$y = (x - 1)^2 - 1$$

$$V(1, -1)$$

c.  $y = 2x^2 + 8x - 3$

$$y = 2[x^2 + 4x] - 3$$

$$y = 2[(x + 2)^2 - 2^2] - 3$$

$$y = 2[(x + 2)^2 - 4] - 3$$

$$y = 2(x + 2)^2 - 8 - 3$$

$$y = 2(x + 2)^2 - 11$$

$$V(-2, -11)$$

d.  $y = x^2 + x - 2$

$$y = \left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 2$$

$$y = \left(x + \frac{1}{2}\right)^2 - \frac{1}{4} - \frac{8}{4}$$

$$y = \left(x + \frac{1}{2}\right)^2 - \frac{9}{4}$$

$$V\left(-\frac{1}{2}, -\frac{9}{4}\right)$$

e.  $y = 3x^2 + 6x - 1$

$$y = 3[x^2 + 2x] - 1$$

$$y = 3[(x + 1)^2 - 1] - 1$$

$$y = 3(x + 1)^2 - 3 - 1$$

$$y = 3(x + 1)^2 - 4$$

$$V(-1, -4)$$

$$\begin{aligned}
 \text{f. } y &= \frac{2}{3}x^2 + \frac{x}{2} - 3 \\
 y &= \frac{2}{3}\left[x^2 + \frac{3}{2} \times \frac{1}{2}x\right] - 3 \\
 y &= \frac{2}{3}\left[x^2 + \frac{3}{4}x\right] - 3 \\
 y &= \frac{2}{3}\left[\left(x + \frac{3}{8}\right)^2 - \frac{9}{64}\right] - 3 \\
 y &= \frac{2}{3}\left(x + \frac{3}{8}\right)^2 - \frac{3}{32} - \frac{96}{32} \\
 y &= \frac{2}{3}\left(x + \frac{3}{8}\right)^2 - \frac{99}{32}
 \end{aligned}$$

18. Solve  $x$  by completing the square:

$$\begin{aligned}
 \text{a. } y &= x^2 + 6x - 3 \\
 y &= (x + 3)^2 - 3^2 - 3 \\
 y &= (x + 3)^2 - 9 - 3 \\
 0 &= (x + 3)^2 - 12 \\
 \text{Add 12 to both sides} \\
 12 &= (x + 3)^2 \\
 \text{Square root both sides} \\
 \pm\sqrt{12} &= x + 3 \\
 -3 \pm \sqrt{12} &= x = -3 \pm 2\sqrt{3} \\
 x_1 &\approx 3.87 \\
 x_2 &\approx -6.46
 \end{aligned}$$

$$\begin{aligned}
 \text{b. } y &= 3x^2 - 6x - 1 \\
 y &= 3[x^2 - 2x] - 1 \\
 y &= 3[(x - 1)^2 - 1^2] - 1 \\
 y &= 3[(x - 1)^2 - 1] - 1 \\
 y &= 3(x - 1)^2 - 3 - 1 \\
 y &= 3(x - 1)^2 - 4 \\
 0 &= 3(x - 1)^2 - 4 \\
 \text{Add 4 to both sides} \\
 4 &= 3(x - 1)^2 \\
 \text{Divide by 3} \\
 \frac{4}{3} &= (x - 1)^2 \\
 \text{Square root both sides} \\
 \pm\sqrt{\frac{4}{3}} &= x - 1 \\
 \pm\frac{\sqrt{4}}{\sqrt{3}} &= x - 1 \\
 \pm\frac{2}{\sqrt{3}} &= x - 1 \\
 \pm\frac{2\sqrt{3}}{3} &= x - 1 \\
 \text{Add 1 to both sides} \\
 1 \pm \frac{2\sqrt{3}}{3} &= x \\
 x_1 &\approx 2.15 \\
 x_2 &\approx -0.15
 \end{aligned}$$

19. Enrichment:

$$\begin{aligned}
 \text{a. Show that } x &= -\frac{b}{2a} \text{ is the equation of the line of symmetry} \\
 ax^2 + bx + c & \\
 y &= ax^2 + bx + c
 \end{aligned}$$

$$y = a \left[ x^2 + \frac{b}{a}x \right] + c$$

$$y = a \left[ \left( x + \frac{b}{2a} \right)^2 - \left( \frac{b}{2a} \right)^2 \right] + c$$

$$y = a \left( x + \frac{b}{2a} \right)^2 - a \left( \frac{b}{2a} \right)^2 + c$$

$$y = a \left( x + \frac{b}{2a} \right)^2 - a \times \frac{b^2}{4a^2} + c$$

$$y = a \left( x + \frac{b}{2a} \right)^2 - \frac{b^2}{4a} + c$$

Notice that the equation of the line of symmetry (or the vertex's x-value) is  $-\frac{b}{2a}$

- b. Show that the roots of a quadratic function are  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  by completing the square on  $y = ax^2 + bx + c$

$$0 = a \left( x + \frac{b}{2a} \right)^2 - \frac{b^2}{4a} + c$$

$$\frac{b^2}{4a} - c = a \left( x + \frac{b}{2a} \right)^2$$

Divide by  $a$

$$\frac{b^2}{4a^2} - \frac{c}{a} = \left( x + \frac{b}{2a} \right)^2$$

$$\frac{b^2}{4a^2} - \frac{4ac}{4a^2} = \left( x + \frac{b}{2a} \right)^2$$

$$\frac{b^2 - 4ac}{4a^2} = \left( x + \frac{b}{2a} \right)^2$$

Square root both sides

$$\pm \sqrt{\frac{b^2 - 4ac}{4a^2}} = x + \frac{b}{2a}$$

Subtract by  $\frac{b}{2a}$

$$-\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} = x$$

$$-\frac{b}{2a} \pm \sqrt{\frac{1}{2a} \times \frac{1}{2a} (b^2 - 4ac)} = x$$

$$-\frac{b}{2a} \pm \frac{1}{2a} \sqrt{b^2 - 4ac} = x$$

$$\frac{1}{2a} (-b \pm \sqrt{b^2 - 4ac}) = x$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = x$$

$$\text{Quadratic Formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

20.  $f(x) = x^2 - 4x$

- a. Solve by factoring

$$y = x(x - 4)$$

$$0 = (x - 0)(x - 4)$$

Either  $x - 0 = 0$  or  $x - 4 = 0$

$$\therefore x = 0 \text{ or } x = 4$$

- b. Solve by using the quadratic formula

$$y = 1x^2 - 4x + 0$$

$$y = ax^2 + bx + c$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(0)}}{2(1)}$$



$$x = \frac{4 \pm \sqrt{16}}{2} = \frac{4 \pm 4}{2}$$

$$x_1 = \frac{4+4}{2} = \frac{8}{2} = 4$$

$$x_2 = \frac{4-4}{2} = \frac{0}{2} = 0$$

c. Solve by completing the square

$$y = x^2 - 4x$$

$$y = (x - 2)^2 - 2^2$$

$$y = (x - 2)^2 - 4$$

$$0 = (x - 2)^2 - 4$$

$$4 = (x - 2)^2$$

Square root both sides

$$\pm\sqrt{4} = x - 2$$

$$\pm 2 = x - 2$$

$$+2 = x - 2 \rightarrow 4 = x$$

$$-2 = x - 2 \rightarrow 0 = x$$

21. Simplify  $\frac{-4 \pm \sqrt{8}}{-2}$

$$= -\frac{4}{-2} \pm \frac{\sqrt{8}}{-2} = 2 \pm \frac{2\sqrt{2}}{-2} = 2 \pm \sqrt{2}$$

22. Solve a quadratic that cannot be factored:

$$y = 3x^2 + 2x - 4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-4)}}{2(3)} = \frac{-2 \pm \sqrt{4 + 48}}{6} = \frac{-2 \pm \sqrt{2 \times 2 \times 13}}{6}$$

$$x = \frac{-2 \pm 2\sqrt{13}}{6} = -\frac{2}{6} \pm \frac{2\sqrt{13}}{6} = -\frac{1}{3} \pm \frac{\sqrt{13}}{3}$$

23. Attempt to solve a quadratic that has no roots:

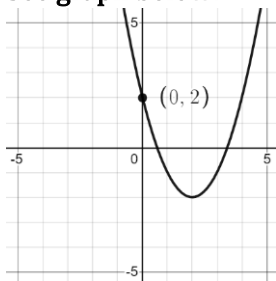
Solve  $y = x^2 + 3x + 3$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(3)}}{2(1)} \dots$$

No solution

24. Find the equation of the parabola:

a. See graph below:



$$y = a(x - b)^2 + c \text{ (Memorize this vertex form of a parabola)}$$

We see that the vertex is  $V(2, -2)$

$$y = a(x - 2)^2 - 2$$

Try to see another point on the graph (0, 2) or (4, 2)

Substitute (0, 2)

$$2 = a(0 - 2)^2 - 2$$

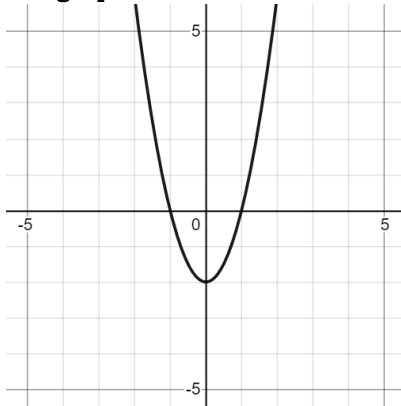
$$4 = a(4)$$

$$1 = a$$

$$y = 1(x - 2)^2 - 2$$

$$y = (x - 2)^2 - 2$$

b. See graph below:



$$y = a(x - b)^2 + c$$

$$y = a(x - 0)^2 - 2$$

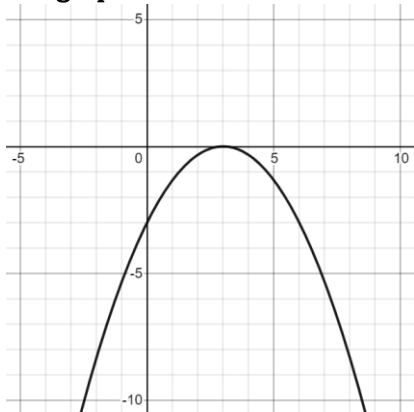
Substitute (1, 0)

$$0 = a(1)^2 - 2$$

$$2 = a$$

$$y = 2x^2 - 2$$

c. See graph below:



$$y = a(x - b)^2 + c$$

$$y = a(x - 3)^2 + 0$$

Substitute (0, -3)

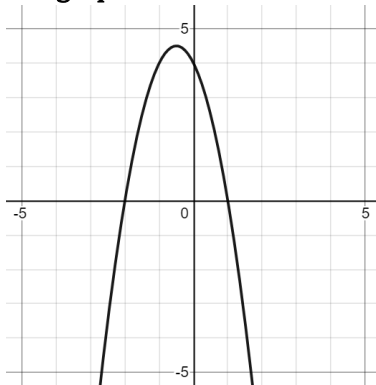
$$-3 = a(0 - 3)^2 + 0$$

$$-3 = 9a$$

$$-\frac{3}{9} = a = -\frac{1}{3}$$

$$y = -\frac{1}{3}(x - 3)^2$$

d. See graph below:



$$y = a(x - b)^2 + c$$

$$y = a\left(x + \frac{1}{2}\right)^2 + \frac{9}{2}$$

Substitute the point (0, 4)

$$4 = a\left(0 + \frac{1}{2}\right)^2 + \frac{9}{2}$$

$$\frac{8}{2} - \frac{9}{2} = \frac{1}{4}a$$

$$-\frac{1}{2} = \frac{a}{4}$$

$$2a = -4$$

$$a = -\frac{4}{2} = -2$$

$$y = -2\left(x + \frac{1}{2}\right)^2 + \frac{9}{2} \text{ which expands to be:}$$

$$y = -2x^2 - 2x + 4$$

Method 2:

This parabola has roots  $x = -2$  and  $x = 1$  so we know that the factors are:

$$(x + 2)(x - 1)$$

Thus  $y = a(x + 2)(x - 1)$ . We don't know the exact  $a$  value yet but we know it must be negative.

Now let's substitute the point (0, 4).

$$4 = a(0 + 2)(0 - 1)$$

$$4 = -2a$$

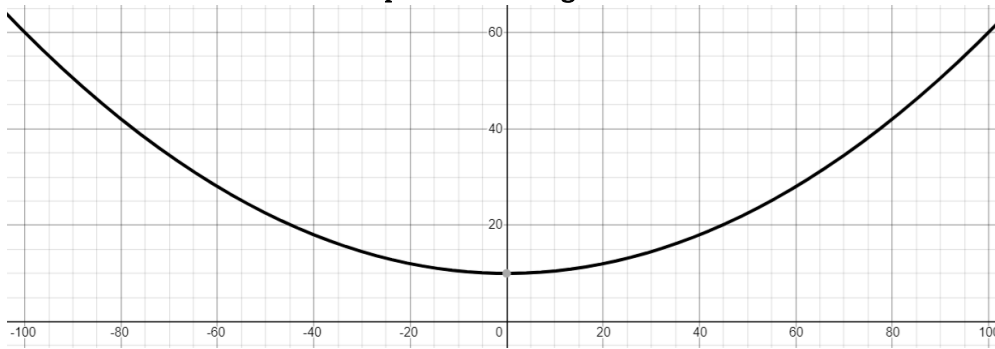
$$a = -2$$

Thus  $y = -2(x + 2)(x - 1)$  or if you expand this out:

$$y = -2(x^2 + x - 2)$$

$$y = -2x^2 - 2x + 4$$

25. Find the exact formula for the parabolic bridge cable below:



$$y = a(x - b)^2 + c$$

The vertex is  $(0, 10)$

$$y = a(x - 0)^2 + 10$$

$$y = ax^2 + 10$$

Substitute the point  $(100, 60)$

$$60 = a(100)^2 + 10$$

$$50 = 10000a$$

$$a = \frac{1}{200}$$

$$y = \frac{1}{200}x^2 + 10$$

26. What is the equation of the parabola that:

a. has a vertex of  $(1, 2)$  and has a y-intercept of 4?

$$y = a(x - b)^2 + c$$

$$y = a(x - 1)^2 + 2$$

Given point  $(0, 4)$

$$4 = a(0 - 1)^2 + 2$$

$$2 = a$$

$$y = 2(x - 1)^2 + 2$$

b. has a vertex of  $(2, 1)$  and contains the point  $(4, -\frac{1}{3})$ ?

$$y = a(x - b)^2 + c$$

$$y = a(x - 2)^2 + 1$$

$$-\frac{1}{3} = a(4 - 2)^2 + 1$$

$$-\frac{1}{3} - \frac{3}{3} = 4a$$

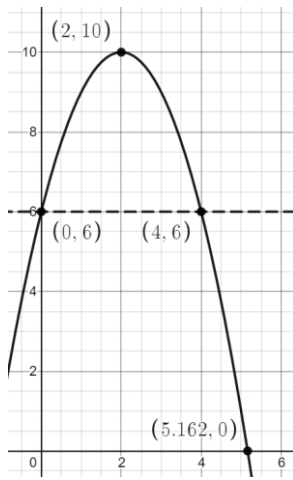
$$-\frac{4}{3} = 4a$$

$$-\frac{4}{12} = a = -\frac{1}{3}$$

$$y = -\frac{1}{3}(x - 2)^2 + 1$$

27. Height (in metres) is a function of time (in seconds). On planet Z,  $h(t) = -(t - 2)^2 + 10$  models your height jumping off a cliff into water.

a. What is your initial height?



6 m (set  $t = 0$ )

b. When do you reach your maximum height?

$$t = 2$$

c. What is the maximum height that you achieve?

$$h = 10 \text{ or } h(2) = 10$$

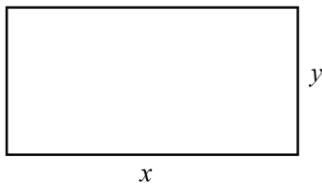
d. After you jump, for how long are you above the height of 6 metres?

4 seconds

e. When do you land in the water?

$$t \approx 5.16$$

28. You have 250 metres of fencing. Find the maximum possible rectangular area.



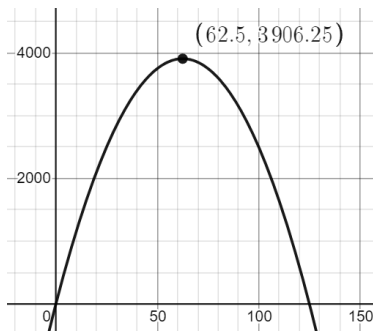
$$2x + 2y = 250$$

$$\text{Divide by 2: } x + y = 125$$

$$\text{Thus, } y = 125 - x$$

$$A = xy$$

$$A = x(125 - x) = -x^2 + 125x$$



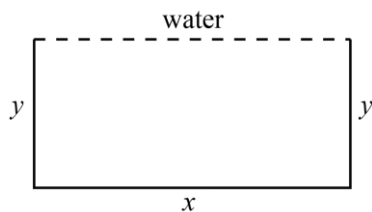
$A$  is a maximized when  $x = -\frac{b}{2a} = -\frac{125}{2(-1)} = 62.5$

When  $x = 62.5$ ,  $y = 125 - x = 125 - 62.5 = 62.5$

Thus,  $A = xy = (62.5)(62.5) = 3906.25 \text{ m}^2$

29. You have 1000 m to fence off your plot of land which is adjacent to a lake. Fencing is only used on three sides of your property because of the water.

a. What dimensions should be used to maximize the area of your land?



$$x + 2y = 1000$$

$$\text{Thus, } 2y = 1000 - x$$

$$y = \frac{1000-x}{2}$$

$$A = xy = x \left( \frac{1000-x}{2} \right)$$

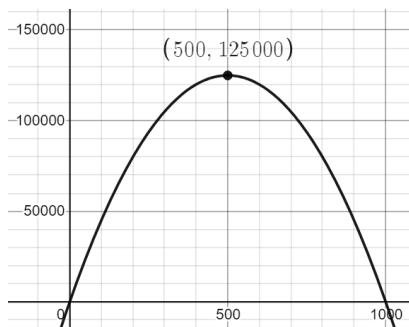
$$A = -\frac{1}{2}x^2 + 500x$$

$$x = -\frac{b}{2a} = \frac{-500}{2\left(-\frac{1}{2}\right)} = 500$$

$$\text{When } x = 500, y = \frac{1000-x}{2} = \frac{500}{2} = 250$$

b. What is the maximum possible area?

$$A = xy = (500)(250) = 125000$$



c. What is the minimum possible area?

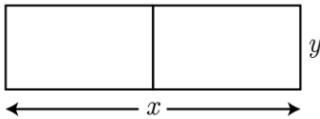
$$\text{When } x = 0, y = \frac{1000-x}{2} = 500$$

$$A = xy = (0)(500) = 0$$

d. In the context of this problem, why is maximizing area not your primary concern?

Having a beach front view may be more important than maximizing area.

30. You have 1200 feet of fencing to enclose two adjacent rectangular regions of equal lengths and widths as shown in the diagram below. What is the maximum area that can be enclosed in the fencing?



$$2x + 3y = 1200$$

$$\text{Thus, } 3y = 1200 - 2x$$

$$y = \frac{1200-2x}{3}$$

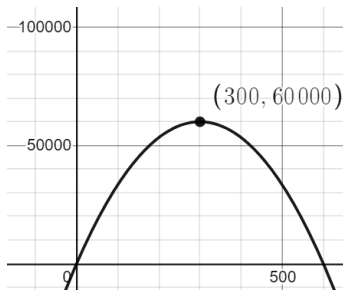
$$A = xy$$

$$A = x \left( \frac{1200-2x}{3} \right)$$

$$A = x \left( \frac{1200}{3} + \frac{-2x}{3} \right)$$

$$A = x \left( -\frac{2}{3}x + 400 \right)$$

$$A = -\frac{2x^2}{3} + 400x$$

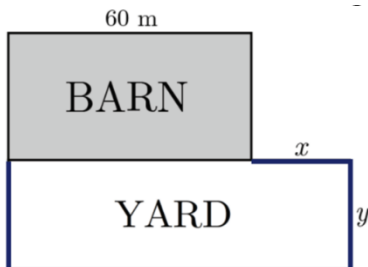


$$x = -\frac{b}{2a} = \frac{-400}{2\left(-\frac{2}{3}\right)} = 300$$

$$\text{When } x = 300, y = 200$$

$$A = xy = (300)(200) = 60\,000 \text{ ft}^2$$

31. You have 300 m of fencing. Find  $x$  and  $y$  to maximize the area of the yard. Only the thick blue border is fenced.



$$A = (x + 60)(y)$$

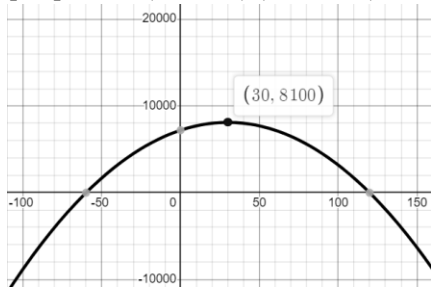
[1]

Visit [hunkim.com/11](http://hunkim.com/11)

$$300 = x + 2y + (x + 60) \quad [2]$$

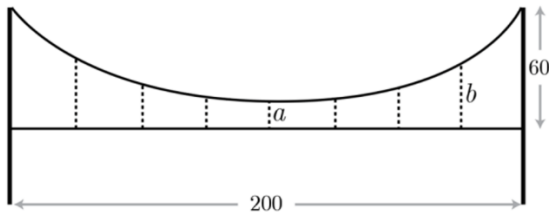
[2a]:  $240 = 2x + 2y \rightarrow x + y = 120 \rightarrow y = 120 - x$   
 Now substitute this  $y$  value into [1]

$$[1a]: A = (x + 60)(120 - x) = -x^2 + 60x + 7200$$



When  $x = 30, y = 120 - x = 90$

32. The shortest cable in the bridge below is  $a = 10$ . Find length  $b$ .



See the vertex of the parabola just above  $a$  at  $(0, 10)$ .

The top right of the this bridge is  $(100, 60)$ .

$$y = a(x - 0)^2 + 10$$

Substitute the point  $(100, 60)$

$$60 = a(100)^2 + 10$$

$$a = \frac{1}{200}$$

$$\text{Thus } f(x) = \frac{1}{200}x^2 + 10$$

The width of each section of the bridge is  $100 \div 4 = 25$

Thus the  $x$ -value of  $b$  is  $x = 75$

$$f(75) = \frac{1}{200}(75)^2 + 10 = 38.125$$

33. You sell 3000 phone cases each month at a price of \$20. For each \$1 price increase, you sell 100 less phone cases.

a. What price should you set to maximize revenue?

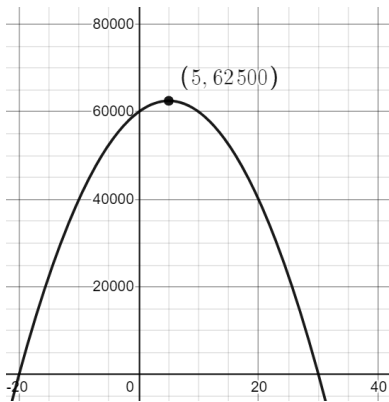
$$\text{Revenue} = \text{price} \times \text{\#items}$$

Let  $x$  be the number of price increases.

$$R = (20 + x)(3000 - 100x)$$

$$R = -100x^2 + 1000x + 60000$$





$$x = -\frac{b}{2a} = \frac{-1000}{2(-100)} = 5$$

When  $x = 5$ , price =  $(20 + x) = \$25$

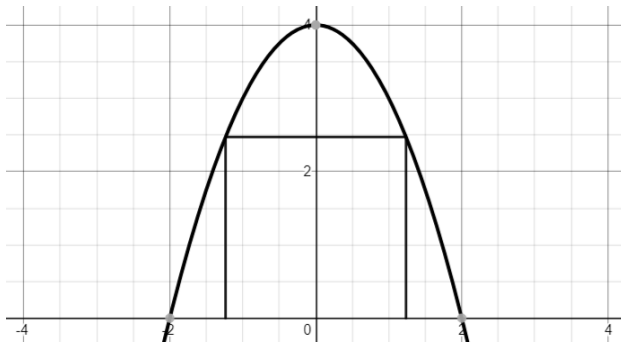
b. What is the maximum revenue?

$$\text{Revenue} = -100(5)^2 + 1000(5) + 60000 = \$62\,500$$

c. How many phone cases are sold when revenue is maximized?

$$\# \text{items} = (3000 - 100x) = 3000 - 100(5) = 2500$$

34. Challenge: A square shaped truck squeezes into the parabola shaped tunnel below. What is the exact value of the maximum width of the truck?



Let  $a$  be the distance from 0 to the right side of the truck. Then the width of the truck is  $2a$ .

Formula of parabola:  $y = -x^2 + 4$

Point on the parabola is  $(x, y) = (x, -x^2 + 4)$

When  $x = a$  the top right point on the square truck is  $(a, -a^2 + 4)$

But we also know that the height of the truck is  $2a$ .

Thus  $2a = -a^2 + 4$  (the y-coordinate of the top right corner of the truck)

$$a^2 + 2a - 4 = 0$$

Using the quadratic formula:

$$a = \sqrt{5} - 1 \text{ (we reject the negative root)}$$

$$\text{Thus the width of the truck is } 2a = 2(\sqrt{5} - 1) = 2\sqrt{5} - 2$$

35. Enrichment - find the discriminant:

a.  $y = x^2 + 2x - 8$

36

$$\text{b. } y = 2x^2 - (k + 1)x + k + 1$$

$$k^2 - 6k - 7$$

36. Challenge: The quadratic curves with equations  $y = x^2 - 4x + 5$  and  $y = m + 2x - x^2$  where  $m$  is a constant, touch once at the point  $P$ . Determine the coordinates of  $P$ .

$$m = \frac{1}{2} \rightarrow x = \frac{3}{2}$$

$$\text{When } x = \frac{3}{2}, y = \frac{5}{4} \text{ i.e. } \left(\frac{3}{2}, \frac{5}{4}\right)$$

Graphs intersect when they are set equal to one another:

$$x^2 - 4x + 5 = m + 2x - x^2$$

$$2x^2 - 6x + 5 - m = 0$$

Touching once means the discriminant  $D = 0$

$$b^2 - 4ac = 0$$

$$(-6)^2 - 4(2)(5 - m) = 0$$

$$36 - 8(5 - m) = 0$$

$$36 - 40 + 8m = 0$$

$$8m = 4$$

$$m = \frac{1}{2}$$

Previously we found that  $2x^2 - 6x + 5 - m = 0$

$$2x^2 - 6x + 5 - \frac{1}{2} = 0$$

$$2x^2 - 6x + \frac{9}{2} = 0$$

$$4x^2 - 12x + 9 = 0$$

$$(2x - 3)^2 = 0$$

$$x = \frac{3}{2}$$

$$\text{When } x = \frac{3}{2}, y = \frac{5}{4}$$