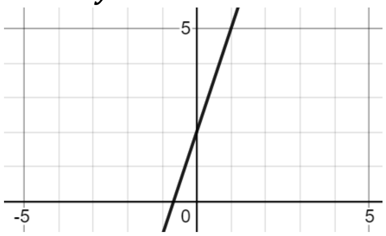
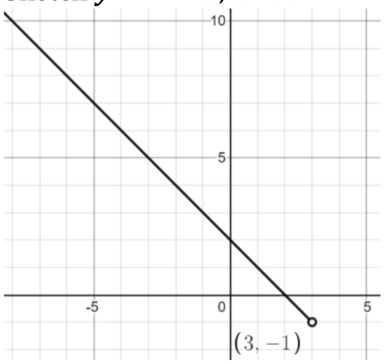
PC11 Quadratic Inequalities Assignment

Name: ______________________

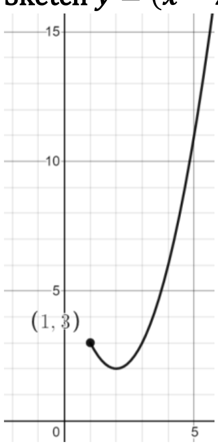
1. **Sketch $y = 3x + 2$**



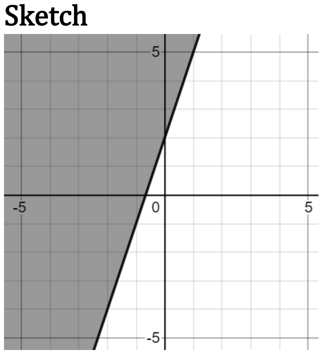
2. **Sketch $y = 2 - x, x < 3$**



3. **Sketch $y = (x - 2)^2 + 2, x \in [1, \infty)$**

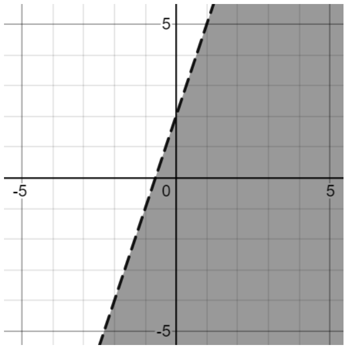


4. **$y \geq 3x + 2$**
   a. **Sketch**



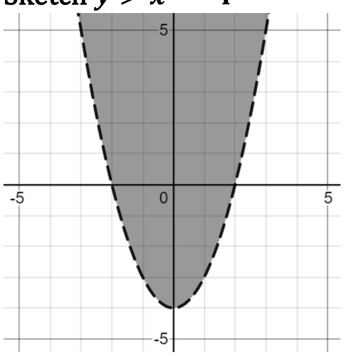
b. Is the point $(1, 7)$ in the solution region?
Yes

5. $y < 3x + 2$
a. Sketch

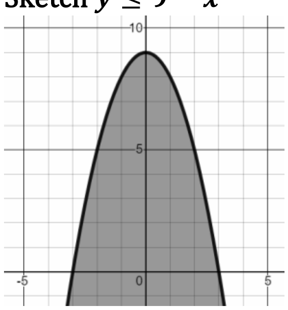


b. Is the point $(-1, -1)$ in the solution region?
No

6. Sketch $y > x^2 - 4$



7. Sketch $y \leq 9 - x^2$



8. Solve $2x = 8$
Divide by 2
$x = 4$

9. Solve $2x < 8$
Divide by 2
$x < 4$

10. Solve $-2x \geq 8$
Method 1:
When you divide (or multiply) by a negative number, make sure that you flip the inequality sign!

Divide by -2
$x \le \frac{8}{-2}$
$x \le -4$

Method 2:
$-2x \ge 8$
Move terms "over the bridge"
$0 \ge 2x + 8$
$-8 \ge 2x$
Now, divide by 2
$-4 \ge x$
$x \le -4$

11. Solve $-0.25x < 3$
Multiply both sides by -4
$x > 3 \times -4$
$x > -12$

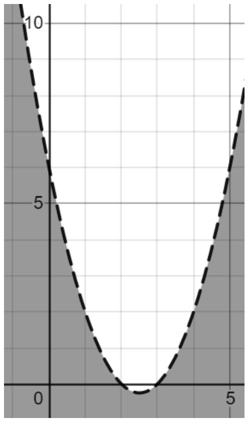
12. Solve $-3x - 9 > x + 4$
$-9 - 4 > x + 3x$
$-13 > 4x$
Divide by 4
$-\frac{13}{4} > x$

13. Solve $-\frac{4}{5} < -\frac{3}{7}a$
Move the terms to make the coefficient of a positive
$\frac{3}{7}a < \frac{4}{5}$
Multiply both sides by $\frac{7}{3}$
$\frac{7}{3} \times \frac{3}{7}a < \frac{4}{5} \times \frac{7}{3}$
$a < \frac{28}{15}$

14. Solve $\frac{2x}{3} + 1 \ge 2(x - 1)$
Multiply both sides by 3
$2x + 3 \ge 6(x - 1)$
$2x + 3 \ge 6x - 6$
$6 + 3 \ge 6x - 2x$
$9 \ge 4x$
Divide by 4
$\frac{9}{4} \ge x$
$x \le \frac{9}{4}$

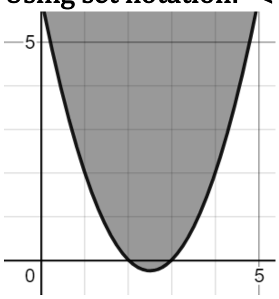
15. Solve $-4 < -2x \le 8$
Divide each part by -2
We flip the inequality sign when we multiply or divide by a negative number
$2 > x \ge -4$ which is equivalent to $-4 \le x < 2$ or $[-4, 2)$

16. Sketch $y < x^2 - 5x + 6$
Either factor or complete the square



17. Solve $x^2 - 5x + 6 \leq 0$

a. Using set notation: $<$ or $\geq$



Factor
$(x-2)(x-3) \leq 0$
$x = 2$ and $x = 3$ are the roots of the quadratic function:
$y = x^2 - 5x + 6$
$2 \leq x \leq 3$

b. Using interval notation: (or [
$x \in [2, 3]$

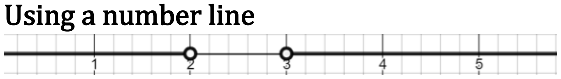
c. Using a number line

18. Solve $x^2 - 5x + 6 > 0$

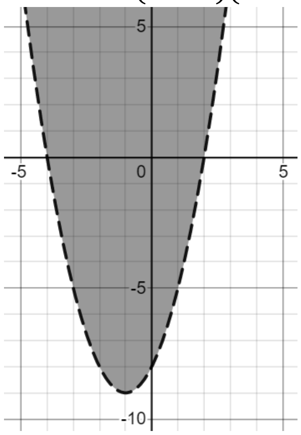
a. Using set notation: $<$ or $\geq$
Again, if you factor you get the critical points $x = 2$ and $x = 3$
$(x-2)(x-3) > 0$
$x < 2$ or $x > 3$

b. Using interval notation: (or [
$x \in (-\infty, 2) \cup (3, \infty)$

c. Using a number line



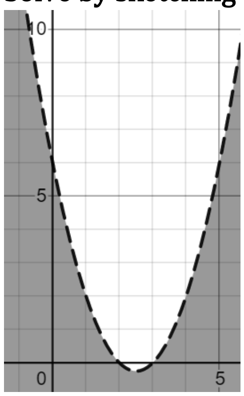
19. Solve $\mathbf{0 > (x-2)(x+4)}$



The critical points are $\mathbf{x = 2, -4}$
However the quadratic is $< \mathbf{0}$ which means:
$\mathbf{-4 < x < 2}$

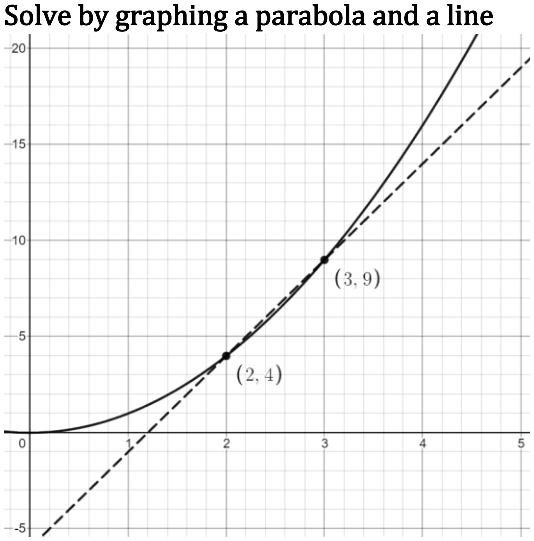
20. $\mathbf{x^2 > 5x - 6}$

a. Solve by sketching a single parabola (and factoring)



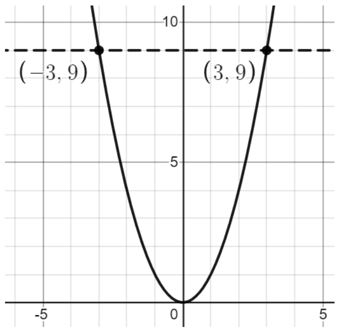
$\mathbf{x^2 - 5x + 6 > 0}$
$\mathbf{(x-2)(x-3) > 0}$
$\mathbf{x < 2}$ or $\mathbf{x > 3}$

b. Solve by graphing a parabola and a line



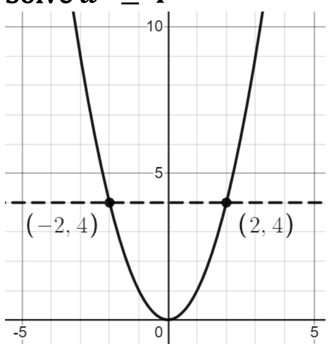
$\mathbf{x^2 > 5x - 6}$
Means when is the $\mathbf{y = x^2}$ graph above the $\mathbf{y = 5x - 6}$ graph?
$\mathbf{x < 2}$ or $\mathbf{x > 3}$

21. Solve $x^2 < 9$



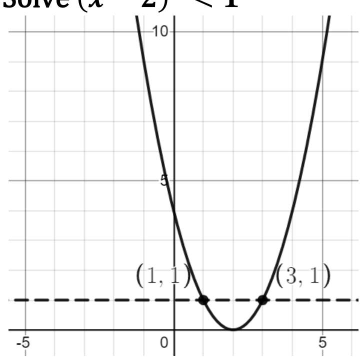
$-3 < x < 3$

22. Solve $a^2 \geq 4$



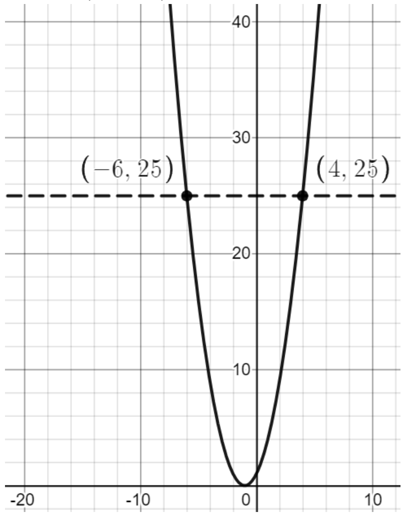
$a \leq -2$ or $a \geq 2$

23. Solve $(x-2)^2 < 1$



$-1 < x - 2 < 1$
Add 2 to each part:
$1 < x < 3$

24. Solve $(x+1)^2 \geq 25$



$x+1 \leq -5$ or $x+1 \geq 5$

$x \leq -6$ or $x \geq 4$

25. Solve $(x-3)^2 - 1 \leq 4$

$(x-3)^2 \leq 5$

$-\sqrt{5} \leq x-3 \leq \sqrt{5}$

$-\sqrt{5}+3 \leq x \leq \sqrt{5}+3$

26. $-\frac{4}{3}(2x-6)^2 + 3 < 0$

a. Solve using algebra

Multiply by 3

$-4(2x-6)^2 + 9 < 0$

$-4(2x-6)^2 < -9$

Divide by -4

$(2x-6)^2 > \frac{9}{4}$

Square root both sides

$2x-6 < -\frac{3}{2}$ or $2x-6 > \frac{3}{2}$

Case 1: $2x-6 < -\frac{3}{2}$

Multiply by 2: $4x-12 < -3$

$4x < 9$

$x < \frac{9}{4}$ or $x < 2.25$

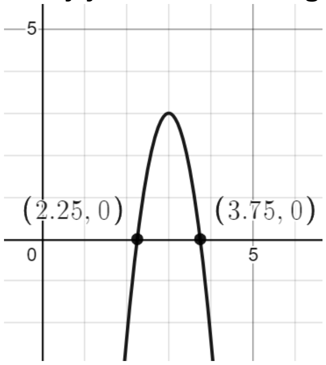
Case 2: $2x-6 > \frac{3}{2}$

$2x > \frac{3}{2} + \frac{12}{2}$

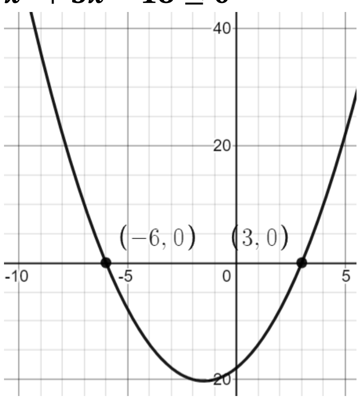
$2x > \frac{15}{2}$

$x > \frac{15}{4}$ or $x > 3.75$

b. Verify your answer using Desmos



27. You need to make a garden which has an area less than 18 m[2].
The length should be 3 m longer than the width.
What are the possible dimensions of the box?
$\boldsymbol{A = x(x+3)}$
$\boldsymbol{x(x+3) \leq 18}$
$\boldsymbol{x^2 + 3x - 18 \leq 0}$



$\boldsymbol{-6 \leq x \leq 3}$
However, there is another restriction: $\boldsymbol{x > 0}$ because a length must be positive.
Thus $\boldsymbol{0 < x \leq 3}$
When $\boldsymbol{x = 0}$, the longer dimension is 3.
When $\boldsymbol{x = 3}$, the longer dimension is 6.