PC11 Radicals Assignment Solutions

1. Evaluate $\sqrt{121}$
   11

2. Solve $a^2 = 9$
   $a = \pm 3$

3. Write as a mixed radical:
   a. $\sqrt{27}$
      $3\sqrt{3}$

   b. $\sqrt{6125}$
      $\sqrt{5^3 \times 7^2}$
      $35\sqrt{5}$

4. Write as an entire radical
   a. $3\sqrt{5}$
      $\sqrt{45}$

   b. $2\sqrt[3]{3}$
      $\sqrt[3]{24}$

   c. $-3\sqrt[3]{2}$
      $-\sqrt[3]{54}$ or $\sqrt[3]{-54}$

5. If possible, evaluate
   a. $\sqrt{-25}$
      Undefined

   b. $\sqrt[3]{-64}$
      -4

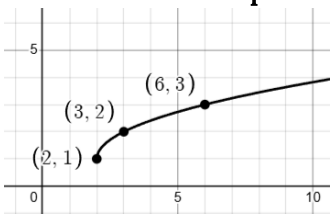
   c. $\sqrt{90000}$
      300

   d. $\sqrt{0.04}$
      0.2

   e. $\sqrt{\frac{1}{361}}$
      $\frac{1}{19}$

6. Order from least to greatest: $\sqrt{16}, 4\sqrt{3}, 5, e$
   $\sqrt{16}, \sqrt{48}, \sqrt{25}, \sqrt{e^2}$
   Note $e \approx 2.7$ thus $e^2 \approx 7.29$
   $e, \sqrt{16}, 5, 4\sqrt{3}$

7. $f(x) = \sqrt{x-2} + 1$
   a. Sketch and label 3 points



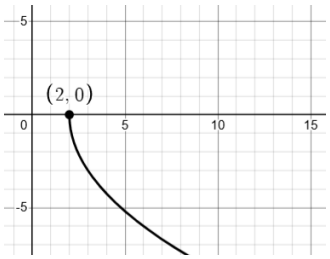
   b. Evaluate $f(27)$
   6

   c. Domain?
   $x \geq 2$

   d. Range?
   $y \geq 1$

8. $y = \sqrt{x-p} + q$. Given $p, q < 0$, describe the transformation.
   ex. $y = \sqrt{x-(-2)} + (-3) = \sqrt{x+2} - 3$
   p units left and q units down

9. $y = -3\sqrt{x-2}$
   a. Sketch and describe the transformation



   b. Domain?
   $x \geq 2$

   c. Range?
   $y \leq 0$

10. $y = a\sqrt{x+b} + c$
    Given $a, b, c < 0$ describe the transformation.
    ex. $y = -2\sqrt{x-3} - 1$
    Multiply y's by a
    Shift b units right
    Shift c units down

11. $y = -\sqrt{x+5}$
    a. Domain?
    $x \geq -5$

    b. Range?
    $y \leq 0$

12. Find the domain of:
    a. $\sqrt{2-7x}$
    $2 - 7x \geq 0$
    $2 \geq 7x$
    $x \leq \frac{2}{7}0$

    b. $\frac{\sqrt{1+3x}}{x^2-1}$
    $\frac{\sqrt{1+3x}}{(x+1)(x-1)}$
    $1 + 3x \geq 0$
    $3x \geq -1$
    $x \geq -\frac{1}{3}$
    $x \neq \pm 1$

    c. $\frac{3\sqrt{x}}{x^2-9x+18}$
    $\frac{3\sqrt{x}}{(x-3)(x-6)}$
    $x \geq 0$
    $x \neq 3, 6$

    d. $\frac{\sqrt{4x-7}}{2x^2+5x-12}$
    $\frac{\sqrt{4x-7}}{(2x-3)(x+4)}$

13. Rationalize:
    a. $\frac{1}{\sqrt{3}}$
    $\frac{\sqrt{3}}{3}$

    b. $\frac{3}{\sqrt{27}}$
    $\frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

    c. $\frac{4}{2-\sqrt{8}}$
    $\frac{4}{2-2\sqrt{2}} = \frac{2}{1-\sqrt{2}} \cdot \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{2(1+\sqrt{2})}{1-2}$
    $-2(1+\sqrt{2})$

d. $\frac{2}{\sqrt[3]{2}}$

$\frac{2}{\sqrt[3]{2}} \times \frac{\sqrt[3]{2}}{\sqrt[3]{2}} \times \frac{\sqrt[3]{2}}{\sqrt[3]{2}}$

$\frac{2\sqrt[3]{4}}{2} = \sqrt[3]{4}$

14. Simplify $\sqrt{8} - 4\sqrt{2}$

$2\sqrt{2} - 4\sqrt{2} = -2\sqrt{2}$

15. Simplify $\sqrt{8} - \sqrt[3]{32} - 2\sqrt{2} + \sqrt[3]{4}$

$2\sqrt{2} - \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2} - 2\sqrt{2} + \sqrt[3]{4}$

$-2\sqrt[3]{4} + \sqrt[3]{4}$

$-\sqrt[3]{4}$

16. Simplify $\frac{-3+\sqrt{27}}{3}$

$-1 + \sqrt{3}$

17. $4\sqrt{3} \cdot 5\sqrt{2}$

$20\sqrt{6}$

18. $\sqrt{3} \cdot \sqrt{5} \cdot \sqrt{7}$

$\sqrt{105}$

19. $a^b \cdot \sqrt{d} \cdot a^c \cdot \sqrt[3]{d}$

$a^{b+c} d^{\frac{1}{2}} \cdot d^{\frac{1}{3}}$

$a^{b+c} \cdot d^{\frac{5}{6}}$

20. Expand and simplify:

a. $3\sqrt{2}(\sqrt{4} + 2\sqrt{3} - 1)$

$2\sqrt{2}(1 + 2\sqrt{3}) = 2\sqrt{2} + 4\sqrt{6}$

b. $(3 - \sqrt{2})^2$

$9 - 6\sqrt{2} + 2$

$11 - 6\sqrt{2}$

c. $(\sqrt{2} - \sqrt{3})(\sqrt{6} + \sqrt{3})$

$\sqrt{12} + \sqrt{6} - \sqrt{18} - 3$

$2\sqrt{3} + \sqrt{6} - 3\sqrt{2} - 3$

d. $-2(\sqrt{8} - \sqrt{2})(1 - \sqrt{8})$

$-2(2\sqrt{2} - \sqrt{2})(1 - 2\sqrt{2})$

$-2\sqrt{2}(1 - 2\sqrt{2})$

$-2\sqrt{2} + 4(2)$

$-2\sqrt{2} + 8$

e. $(\sqrt{27}-1)^3$

$(3\sqrt{3}-1)(3\sqrt{3}-1)^2$

$(3\sqrt{3}-1)(9(3)-6\sqrt{3}+1)$

$(3\sqrt{3}-1)(28-6\sqrt{3})$

$84\sqrt{3}-18\sqrt{3}-28+6\sqrt{3}$

$72\sqrt{3}-28$

f. $(\sqrt{3}-\sqrt{2})(1+\sqrt{5}+3)$

$(\sqrt{3}-\sqrt{2})(4+\sqrt{5})$

$4\sqrt{3}+\sqrt{15}-4\sqrt{2}-\sqrt{10}$

21. A rectangle has a base of $5\sqrt{2}-3\sqrt{3}$ and a height of $2\sqrt{8}-\sqrt{3}$

a. Area in simplified form?

$(5\sqrt{2}-3\sqrt{3})(4\sqrt{2}-\sqrt{3})$

$20(2)-5\sqrt{6}-12\sqrt{6}$

$40-17\sqrt{6}$

b. Perimeter in simplified form?

$2(5\sqrt{2}-3\sqrt{3})+2(4\sqrt{2}-\sqrt{3})$

$10\sqrt{2}-6\sqrt{3}+8\sqrt{2}-2\sqrt{3}$

$18\sqrt{2}-8\sqrt{3}$

22. A cylinder has a diameter of $\sqrt{125}$ and a height of 100

a. Volume?

$d=5\sqrt{5}$

$r=\frac{5}{2}\sqrt{5}$

$V=\pi r^2 h=\pi\left(\frac{5\sqrt{5}}{2}\right)^2\cdot 100$

$100\pi\cdot\frac{25(5)}{4}$

3125π units^3

b. Area including the bottom?

$A=2\pi r^2+\pi dh$

$2\pi\left(\frac{5\sqrt{5}}{2}\right)^2+\pi(5\sqrt{5})(100)$

$25(5)\pi+500\pi\sqrt{5}$

$50\pi+500\pi\sqrt{5}$ or $50\pi(1+10\sqrt{5})$

23. Solve:

a. $\sqrt{x}=9$

$x=81$

b. $3\sqrt{x}=5$

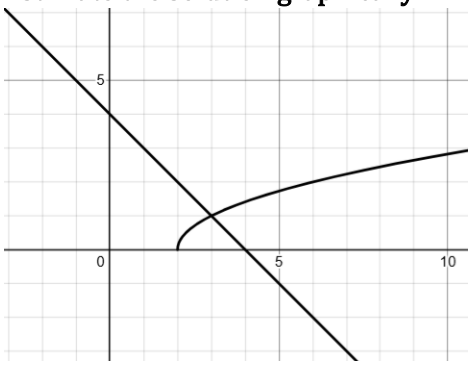
$9x=25$

$x = \frac{25}{9}$

c. Solve $\sqrt{x+1} = 3$
$x + 1 = 9$
$x = 8$

24. $\sqrt{x-2} = 4 - x$

a. Estimate the solution graphically



b. Find the point of intersection algebraically
$x - 2 = (4 - x)^2$
$x - 2 = 16 - 8x + x^2$
$0 = x^2 - 9x + 18$
$0 = (x - 3)(x - 6)$
$x = 3$ (reject $x = 6$)

c. Check for extraneous roots
$x = 6$ is extraneous
$LS = \sqrt{6-2} = \sqrt{4} = 2$
$RS = 4 - 6 = -2$
$LS \neq RS$ $\therefore$ reject $x = 6$

d. Find the point of intersection
When $x = 3$, $y = 4 - x = 4 - 3 = 1$
$(3, 1)$

25. Solve $\sqrt{x+3} = 5 - \frac{1}{3}x$
$3\sqrt{x+3} = 15 - x$
$9(x+3) = 225 - 30x + x^2$
$9x + 27 = 225 - 30x + x^2$
$0 = x^2 - 39x + 198$
$0 = (x - 6)(x - 33)$
$x = 6$ (reject $x = 33$)

26. Define: $|a|$
$a = \begin{cases} a & a \geq 0 \\ -a & a < 0 \end{cases}$

27. Simplify $\sqrt{x^2y^4}$

$|x| \cdot y^2$