PC11 Rational Exponents Lesson Solutions

Powers is another topic in the BC Pre-Calculus 11 curriculum. Although you may have learned about exponent laws since Grade 9, this is the first year that BC math students fully learn the topic of powers which include negative exponents and fractional exponents.

- Positive and negative rational exponents
- Exponent laws
- Evaluation using order of operations
- Numerical and variable bases

1. 2^{10}
   1024

2. $(-2)^4$
   16

3. -2^4
   -16

4. $0^1 + 1^0$
   $0 + 1 = 1$

5. Enrichment: 0^0
   Undefined

6. $-3(-3)^2$
   $-3 \times 9 = -27$

7. $(-1)^{1234}$
   1

8. $3\left(\frac{2}{3}\right)^2$
   $3 \times \frac{4}{9} = \frac{4}{3}$

9. 2^{-3}
   $\frac{1}{8}$

10. ab^{-c}
    $\frac{a}{b^c}$

11. $\left(\frac{3}{5}\right)^{-3}$
    $\left(\frac{5}{3}\right)^3 = \frac{125}{27}$

12. $\frac{a^3}{a^{-2}}$
    a^5

13. $\dfrac{x(x^5)^2(x^3)}{x^{-1}}$

$\dfrac{x^{14}}{x^{-1}} = x^{15}$

14. $\left(\dfrac{8a^3b^5}{4a^2b^7}\right)^{-3}$

$\left(\dfrac{4a^2b^7}{8a^3b^5}\right)^3$

$\left(\dfrac{b^2}{2a}\right)^3 = \dfrac{b^6}{8a^3}$

15. $\dfrac{2x^2y(3xy)^{-3}}{(4xy^{-3})^2}$

$\dfrac{2x^2y}{16x^2y^{-6}(3xy)^3}$

$\dfrac{y^7}{8\times27x^3y^3} = \dfrac{y^4}{216x^3}$

16. Write $\sqrt{8}$ in the form $a\sqrt{b}$

$2\sqrt{2}$

17. Write $\sqrt{x}$ as a fractional exponent

$x^{\frac{1}{2}}$

18. $9^{\frac{1}{2}}$

3

19. $x^{\frac{a}{b}} = \sqrt[?]{?}$

$\sqrt[b]{x^a}$

20. Write $x^{\frac{3}{2}}$ as a mixed radical

$x\sqrt{x}$

21. $(0.09)^{1/2}$

0.3

22. $(-8)^{\frac{1}{3}}$

-2

23. $\sqrt[3]{\sqrt{x}} = x^k$. Find k

$x^{\frac{1}{6}} \to k = \dfrac{1}{6}$

24. True or False:

a. $\sqrt{9} = \pm3$

False

b. Given $x^2 = 9$, $x = \pm3$

True

25. Solve $a^2 = 100$

± 10

26. Solve $32 = w^4 + w^4$

$32 = 2w^4$

$16 = w^4$

$w = \pm 2$

27. Solve $x^4 = -16$

No solution

28. $125^{-\frac{2}{3}}$

$\left(5^3\right)^{-\frac{2}{3}} = 5^{-2} = \frac{1}{25}$

29. Show that $1^{-2.5} = 1$

$LS = \dfrac{1}{1^{2.5}} = \dfrac{1}{1^{\frac{5}{2}}} = \dfrac{1}{\sqrt{1^5}} = \dfrac{1}{\sqrt{1}} = \dfrac{1}{1} = 1 = RS$

30. Convert $\sqrt{8}$ to a mixed radical

$2\sqrt{2}$

31. Convert $\sqrt[3]{243}$ to a mixed radical

$\sqrt[3]{3^5}$

$3\sqrt[3]{9}$

32. Convert $3\sqrt{2}$ to an entire radical

$\sqrt{18}$

33. Convert $2\sqrt[5]{3}$ to an entire radical

$\sqrt[3]{96}$

34. Simplify $x^2 \times x^{1/2}$ and express as a mixed radical in the form $a\sqrt{b}$

$x^{\frac{5}{2}} = \sqrt{x^5} = x^2\sqrt{x}$

35. Simplify $\dfrac{x}{x^{2/3}}$. Express your answer in the form $x^{\frac{a}{b}}$

$x^{\frac{1}{3}}$

36. Simplify $\left(\sqrt[3]{x^2}\right)\left(\sqrt[4]{x^5}\right)$ using fractional exponents in the form $x^{a/b}$

$x^{\frac{2}{3}} \cdot x^{\frac{5}{4}} = x^{\frac{23}{12}}$

37. Write $\dfrac{15^6}{3^6}$ as a single power

$\left(\dfrac{15}{3}\right)^6 = 5^6$

38. Solve $\dfrac{3^{10}}{3^x} = 3^6$

$3^{10-x} = 3^6$

$$10 - x = 6$$
$$4 = x$$

39. Solve $25^{2x-3} = 125^{2-5x}$

$$(5^2)^{(2x-3)} = (5^3)^{(2-5x)}$$
$$4x - 6 = 6 - 15x$$
$$19x = 12$$
$$x = \frac{12}{19}$$

40. Solve $x^{3/5} = 2$

$$x = 2^{\frac{5}{3}}$$

41. Solve $x^{2/3} = 4$

$$x = \pm 4^{\frac{3}{2}} = \pm(2^2)^{\frac{3}{2}} = \pm 8$$

42. Solve $\left(\frac{2^{1-2x}}{2^{x+3}}\right)^3 = 4$

$$(2^{-3x-2})^3 = 4$$
$$2^{-9x-6} = 2^2$$
$$-9x - 6 = 2$$
$$-8 = 9x$$
$$x = -\frac{8}{9}$$

43. Simplify $\frac{(0.6x^{-1})^{-2}}{\left(\frac{2}{x}\right)^3}$

$$\frac{\left(\frac{3}{5x}\right)^{-2}}{\frac{8}{x^3}}$$
$$\frac{25x^2}{9} \times \frac{x^3}{8} = \frac{27}{72}x^5$$

44. Evaluate $\frac{(-2)^{100}}{-2^{97}} + 8^{-\frac{1}{3}} \div 2$

$$-2^3 + (2^3)^{-\frac{1}{3}} \times \frac{1}{2}$$
$$-8 + \frac{1}{2} \times \frac{1}{2}$$
$$-\frac{31}{4}$$

45. Simplify $\left(\frac{-16x^{-2}y}{2x^{-3}y^3}\right)^{-\frac{2}{3}}$ using positive fractional exponents.

$$\left(\frac{(-2)^3 x}{y^2}\right)^{-\frac{2}{3}}$$
$$\left(\frac{y^2}{(-2)^3 x}\right)^{\frac{2}{3}}$$
$$\frac{y^{\frac{4}{3}}}{4x^{\frac{2}{3}}}$$

46. Enrichment: Show that $\sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a$

$$LS = (x^a)^{\frac{1}{b}} = x^{\frac{a}{b}}$$

$$RS = \left(x^{\frac{1}{b}}\right)^a = x^{\frac{a}{b}} = LS$$

47. Challenge: $4^x = 9^y = 6$. Find $\frac{1}{x} + \frac{1}{y}$?

$$4^x = 6$$

$$(4^x)^{\frac{1}{x}} = 6^{\frac{1}{x}}$$

$$4 = 6^{\frac{1}{x}} \qquad \text{Equation 1}$$

$$9^y = 6$$

$$(9^y)^{\frac{1}{y}} = 6^{\frac{1}{y}}$$

$$9 = 6^{\frac{1}{y}} \qquad \text{Equation 2}$$

Multiplying Eq 1 and Eq 2:

$$36 = 6^{\frac{1}{x}+\frac{1}{y}}$$

$$6^2 = 6^{\frac{1}{x}+\frac{1}{y}}$$

Thus $\frac{1}{x} + \frac{1}{y} = 2$