PC11 Trigonometry Lesson

Last year you learned about right-angle trigonometry: SOH CAH TOA. This year in Pre-Calculus 11 you will learn how to solve non right-angle triangles using the Sine Law and the Cosine Law. Do your best to understand this year's trigonometry concepts because you will learn more about trigonometry next year.

- Use of sine and cosine laws to solve non-right triangles, including ambiguous cases
- Contextual and non-contextual problems
- Angles in standard position
- Degrees
- Special angles, as connected with the 30-60-90 and 45-45-90 triangles
- Unit circle
- Reference and co-terminal angles
- Terminal arm
- Trigonometric ratios
- Simple trigonometric equations

1. Label the location of the four quadrants

2. In which Quadrant is θ located?
   a. $\theta = 120°$

   b. $\theta = -45°$

   c. $\theta = 400°$

   d. $\theta = -1100°$

3. $\theta = 300°$
   a. What is the reference angle?

   b. Find a positive coterminal angle to $\theta = 300°$

   c. Find a negative coterminal angle to $\theta = 300°$

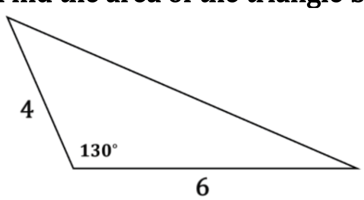
4. Enrichment: Radians vs. Degrees
   This year we measure the angle $\boldsymbol{\theta}$ in degrees. Next year we use a different unit called radians.
   One full revolution $= \boldsymbol{2\pi}$ radians $= \mathbf{360^\circ}$
   a. Convert $\boldsymbol{\pi}$ radians to degrees

   b. Convert $\frac{\pi}{4}$ radians to degrees

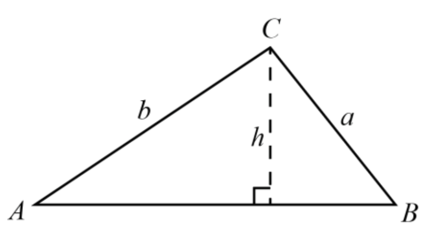
   c. Convert $\frac{\pi}{6}$ radians to degrees

5. Enrichment:
   a. Show that the area of a triangle is $A_\Delta = \frac{1}{2} ab \sin C$

   b. Find the area of the triangle below:

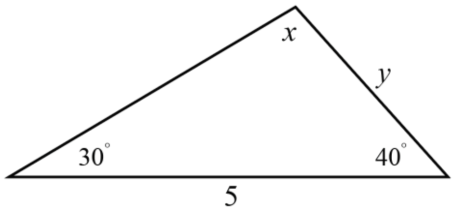


6. Enrichment:
   a. Use the triangle below to help you prove the Sine Law: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

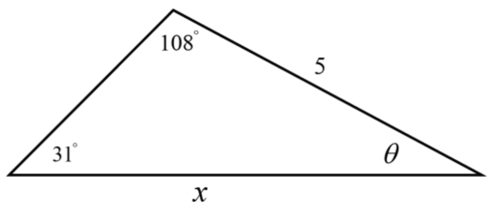


   b. Given the previous proof, why does it follow that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$?

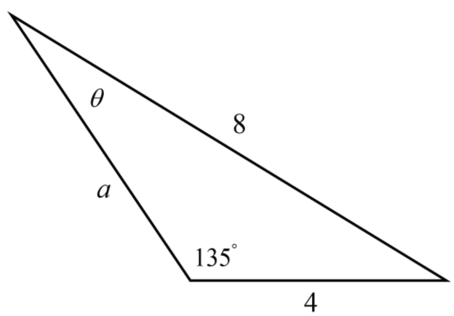
7. Solve x and y the ASA triangle below:



8. Solve x and θ in the following AAS triangle below:

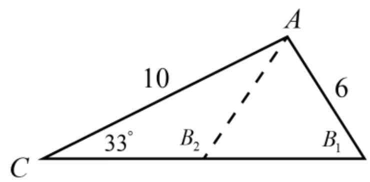


9. Solve θ and a in the following SSA triangle below:



10. No diagram: Solve the following triangle:
$\angle C = 140^\circ, b = 6, c = 30$

11. Consider the ambiguous case:
    $\angle C = 33°$. Side $c = 6$. Side $b = 10$.
    a. What are the possible angles of B?

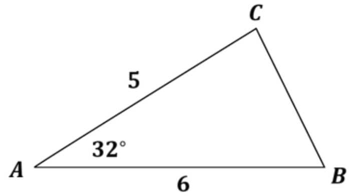


    b. What are the possible lengths of a?

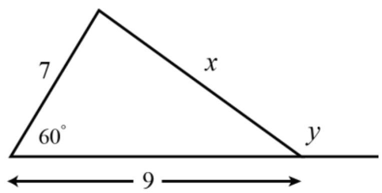
12. Enrichment: State the number of possible triangles that can be formed. Confirm your answer with an online triangle calculator.
    a. $\angle B = 32°, a = 27, b = 22$
    b. $\angle B = 96°, b = 25, a = 6$
    c. $\angle B = 34°, a = 23, b = 7$
    d. $\angle A = 30°, AC = 8, BC = 5$

13. When solving a non-right-angled triangle, when should the Sine Law vs. Cosine Law be used?

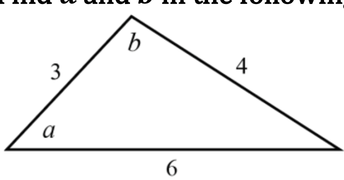
14. Find side length CB in the diagram below:



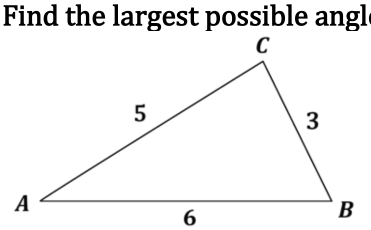
15. Find x and y in the following SAS triangle:



16. Find a and b in the following SSS triangle:

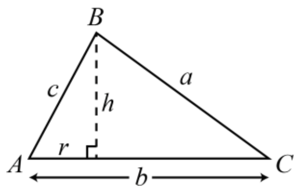


17. Find the largest possible angle in the diagram below:



18. Given $c^2 = a^2 + b^2 - 2ab\cos C$, find an expression for $\angle C$

19. Enrichment: Use your knowledge of the primary trigonometric ratios and the Pythagorean Theorem to prove the Cosine Law:
$c^2 = a^2 + b^2 - 2ab\cos C$



20. Unit circle:
   a. Equation of the unit circle?

   b. Enrichment: What is the equation of a circle with radius r centered at the origin?

   c. Sketch the unit circle

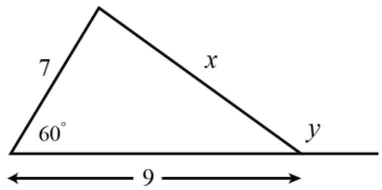
   d. Explain why $y = \sin\theta$ on the unit circle

   e. Explain why $x = \cos\theta$ on the unit circle

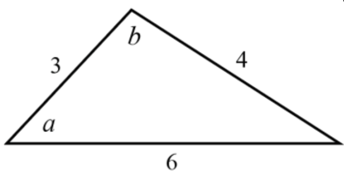
   f. Enrichment: Where does the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$ come from?

21. When solving a non-right-angled triangle, when should the Sine Law vs. Cosine Law be used?

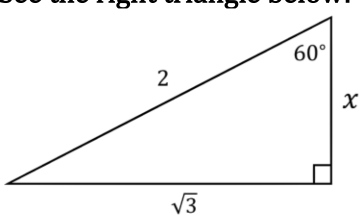
22. Find x and y in the following SAS triangle:



23. Find a and b in the following SSS triangle:



24. See the right triangle below:



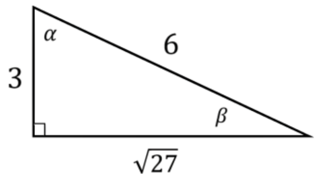
a. Solve x using the Pythagorean Theorem

b. Find the value of the missing angle

c. Solve x using the Sine Law

d. Solve x using the Cosine Law

25. Solve the unknown angles in the diagram below:

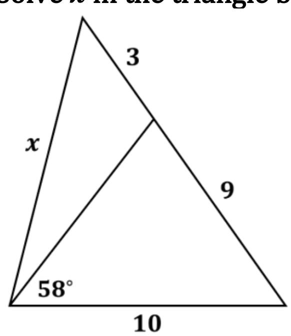


a. Using SOH CAH TOA

b. Using the Sine Law

c. Using the Cosine Law

26. Solve x in the triangle below:

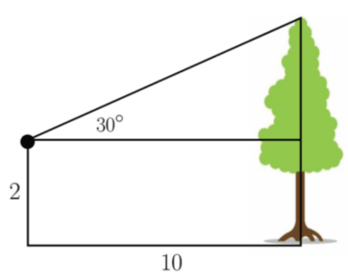


27. Basic trigonometric identity: $\tan\theta = \frac{\sin\theta}{?}$

28. Explain how drawing a 2-2-2 equilateral triangle can help you memorize the primary trigonometric ratios.

29. Memorize the values of the following special angles:
   a. $\sin 30^\circ$
   b. $\sin 45^\circ$
   c. $\sin 60^\circ$
   d. $\cos 30^\circ$
   e. $\cos 45^\circ$
   f. $\cos 60^\circ$
   g. $\tan 30^\circ$
   h. $\tan 45^\circ$
   i. $\tan 60^\circ$

30. Find the exact height of the tree without a calculator and simplify your answer using your knowledge of special angles.



31. Evaluate
   a. $\sin 120°$
   b. $\cos 330°$
   c. $\sin 225°$
   d. $-\sin 225°$
   e. $\tan(-420°)$

32. Quadrantal angles – Find:
   a. $\sin 90°$
   b. $\cos 180°$
   c. $\sin(-360°)$
   d. $\tan(180°)$

33. Label the (x, y) coordinates on the unit circle for $P(\theta)$ when:
   a. $\theta = 30°$
   b. $\theta = 45°$
   c. $\theta = 60°$
   d. $\theta = 90°$
   e. $\theta = 210°$
   f. $\theta = 270°$
   g. $\theta = 315°$
   h. $\theta = 720°$

34. If $\sin\theta$ is negative and $\cos\theta$ is positive, what quadrant must θ be in?

35. Solve the following trigonometric equations within the domain $0 \leq \theta \leq 360°$:

a. $\sin\theta = \frac{1}{2}$

b. $\sin\theta = -\frac{1}{\sqrt{2}}$

c. $\sin A = \frac{\sqrt{2}}{2}$

d. $\sin\beta = -\sqrt{3}/2$

e. $\cos\theta = -0.5$

f. $\tan x = \sqrt{3}$

g. $\tan\theta = -2$

h. $\sin\theta = 2$

36. θ in standard position on the unit circle has coordinates $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$. Find θ

37. Challenge: A boat travels 13 km in the direction $N30°W$.
It then adjusts its course and heads $S70°W$, travelling another 20 km in this new direction.

a. How far is the boat from its initial position?

b. Enrichment: Bearings are angles measured in a clockwise direction from the north line. What is the bearing of the boat in its final position as compared to its initial position?

38. Enrichment: Visually represent $\tan\theta$ on the unit circle.