

PC11 Lesson 1: Types of Numbers (solutions)

Types of Numbers is the first topic of the BC Pre-Calculus 11 curriculum.

The symbol for integers \mathbb{Z} is based on the German word "Zahlen."

BC students do not have to worry about complex numbers but BC math students who are interested in studying in the US should learn that $i = \sqrt{-1}$.

- Real number classification
1. Provide the mathematical symbol and examples of the following types of numbers:
 - a. Natural
 $x \in \mathbb{N}$
ex. 1, 2, 3, 4 (positive whole numbers)
 - b. Whole
 $x \in \mathbb{W}$
ex. 0, 1, 2, 3, 4 (includes 0)
 - c. Integers
 $x \in \mathbb{Z}$
ex. ... - 3, -2, -1, 0, 1, 2, 3, ...
 $x \in \mathbb{Z}^-$ refers to negative integers only
 - d. Rational
 $x \in \mathbb{Q}$
An number that can be expressed in the form $\frac{a}{b}$ where $a, b \in \mathbb{Z}$
ex. $\frac{3}{4}, 2, -0.2, \sqrt{9}$
 - e. Real
 $x \in \mathbb{R}$
ex. 5, $\sqrt{3}, \pi$
 - f. Enrichment: What is a complex number??
 $a + bi$ (a number that consists of a real number plus an imaginary number)
 i is defined to be $\sqrt{-1}$
 2. Rational or irrational?
 - a. $\sqrt{5}$
Irrational
 - b. $\sqrt{16}$
Rational
 - c. $\sqrt{\frac{25}{9}}$
 $= \frac{5}{3}$
Rational

d. π
Irrational

e. 0
 $= \frac{0}{2}$
Rational

f. $1.\bar{6}$
 $1 + \frac{2}{3} = \frac{5}{3}$
Rational

g. 0.02
 $= \frac{2}{100}$
Rational

h. 2.5
 $\frac{2.5}{1} = \frac{25}{10}$
Rational

i. $\sqrt{1.21}$
 $\frac{1.1}{1} = \frac{11}{10}$
Rational

j. $e \approx 2.718$
Irrational

2. Show that 0.4 is rational.

$$0.4 = \frac{4}{10}$$

Which is the form $\frac{a}{b}$ where $a, b \in \mathbb{Z}$

3. Show that $\sqrt{\frac{121}{4}}$ is rational.

$$= \frac{11}{2}$$

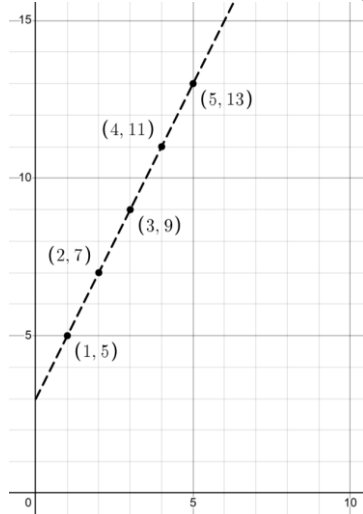
Which is the form $\frac{a}{b}$ where $a, b \in \mathbb{Z}$

4. Rank from least to greatest: 2.5, $\sqrt{9}$, -100, $\frac{8}{3}$, $3.\bar{3}$, ∞ , 200%

$$2.5, 3, -100, 2.\bar{6}, 3.\bar{3}, \infty, 2$$

$$-100, 200\%, 2.5, \frac{8}{3}, \sqrt{9}, 3.\bar{3}, \infty$$

5. Sketch $y = 2x + 3$, $x \in \mathbb{Z}^+$ (positive integer)



Technically we should not include the dashed line

6. Enrichment:

a. List the four prime numbers

2, 3, 5, 7

b. List the first four positive perfect squares

1, 4, 9, 16

c. List the first four positive perfect cubes

1, 8, 27, 64

d. Challenge: 1, 4, 9, 16, 25 ...

What is the value of the n^{th} term?

n^2

7. Challenge:

a. Show that $1.\overline{23}$ is a rational number

Let $x = 0.\overline{23}$

$100x = 23.232323 \dots$

$x = 0.2323232323 \dots$

Subtracting $99x = 23$

$x = \frac{23}{99}$

b. Challenge: Show that $2.\overline{051}$ is a rational number

Focus on $0.\overline{051}$ part

$x = 0.05151515151 \dots$

$1000x = 51.515151 \dots$

$10x = 0.515151 \dots$

Subtracting $990x = 51$

$x = \frac{51}{990}$

Thus $2 + x = \frac{677}{330}$ which is in the form of a rational number