PC12 Polynomial Functions Lesson

Name: _____

- Polynomials functions and equations
- Factoring, including the factor theorem and the remainder theorem
- Graphing and the characteristics of a graph (e.g., degree, extrema, zeros, end-behaviour)
- Solving equations algebraically and graphically

1. What is a polynomial function?

2. Polynomial functions are smooth and c_____.

3. Sketch the quadratic (and polynomial) $f(x) = -2x^2$ and label 3 points.

4. Sketch the cubic polynomial $f(x) = (x-2)^3$ and label 3 points.

5. Sketch the polynomial: $y = (x-2)(x+6)(x+2)$

6. Is $P(x) = \pi x^5 - \sqrt{2}x^2 + 1.\overline{3}$ a polynomial function?

7. Is $P(x) = x^3 - 2x + \frac{1}{x}$ a polynomial function?

8. $f(x) = (2x+1)(x-2)(x+4)^{\frac{2}{3}}$
   a. Find the x-intercepts.

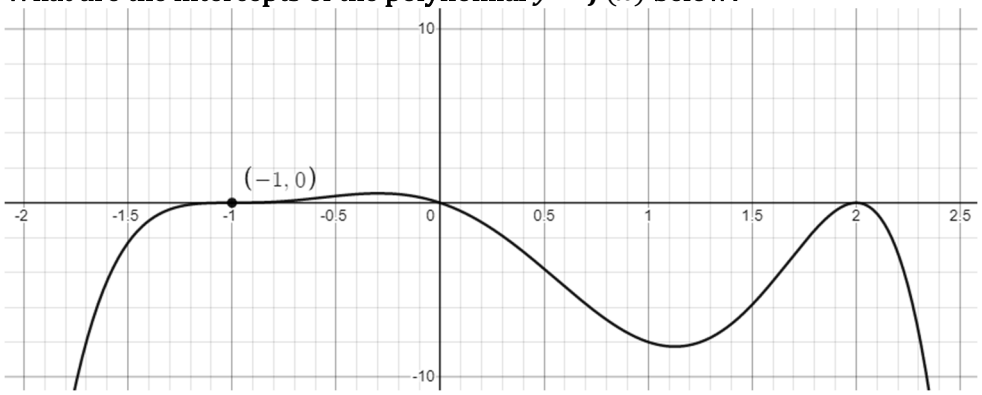
   b. Is $f(x)$ a polynomial?

9. Factor to find the roots of $2x^3 + x^2 - 6x$

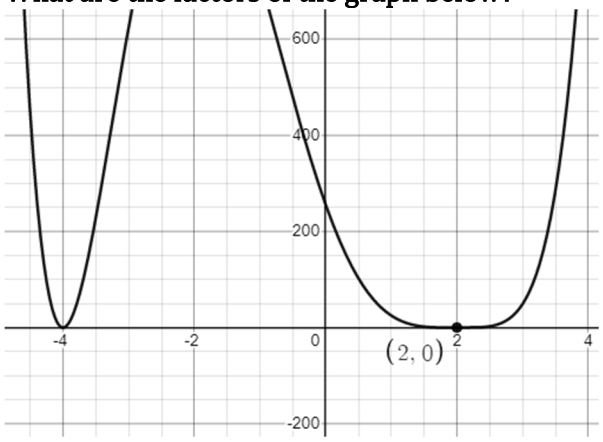
10. Factor to solve when you land in the water $h(t) = 8 - 18t^2$

11. Given $P(x) = (3x - 2)(x + \pi)^2$ what are the x-intercepts?

12. What are the intercepts of the polynomial $y = f(x)$ below?



13. What are the factors of the graph below?



14. Solve $2x^2 = 4x$

15. $P(x) = x(x - 2)^2(x + 4)^3(x - 4)^2$

    a. Evaluate $P(3)$

    b. Intercepts?

c. Describe the end-behavior of the graph as $x \to \infty$ and $x \to -\infty$

d. Sketch the polynomial $P(x)$ while clearly showing the shape (based on the degree of the factors)

16. Sketch $f(x) = -(x-2)(x+6)^2(x)(x-4)^3$

17. Identify the degree of the polynomial $Q(x)$ below:
$P(x) = 16x^5 - 44x^6 + x^8 + 640x^4 - 512x^3 - 3072x^2 + 4096x$

18. Enrichment: What is the degree of the following polynomial: $y = 5x^2y^3 + 4x - 7$

19. $P(x) = (x-2)^2(x+6)^3$
What is the significance of the multiplicity (exponent of the factor) being 2 (or even)?

20. What is the significance of the multiplicity (exponent of the factor) being 3 (or odd)?

21. $P(x) = (x-3)^1(x+2)^1x^1$
What is the significance of the multiplicity (exponent of the factor) being 1?

22. State the Factor Theorem

23. State the Remainder Theorem

24. State the rational root theorem (also called the Rational Zero Theorem)

25. $P(x) = x^3 + 2x^2 - 4x - 8$.

a. Factor by grouping

b. Factor using the rational root theorem and then sketch.

26. $P(x) = -x^4 + 6x^2 + 8x + 3$. Fully factor and sketch this polynomial given $(x - 3)$ is a factor.

27. Although you are welcome to use the Factor Theorem to factor polynomials, sometimes its faster to try to pull out a GCF first. Fully factor and sketch: $y = 4x^3 - 14x^2 + 12x$

28. $\frac{x^3-2x^2+3x+2}{x+2}$

a. Use long division to find the remainder.

b. Use synthetic division to find the remainder.

c. Use the remainder theorem to find the remainder.

d. Express in the form Quotient $+\frac{\text{remainder}}{\text{divisor}}$

29. $P(x) = \frac{x^3-2x+3}{x-1}$. Find the remainder using long division or synthetic division

30. Factor and sketch: $y = 2x^3 + x^2 - 4x - 3$

31. Factor and sketch: $y = x^4 - 2x^3 - 32x^2 + 96x$

32. Enrichment: $P(x) = (2x - 3)^2(3x - 1)$

a. Identify the roots of this polynomials and sketch.

b. Expand this polynomial.

c. Factor this polynomial using the Rational Root Theorem.

33. Fractional roots: Factor $12x^4 - 20x^3 + 11x^2 - 2x$

34. **What is the equation of the mystery degree 4 function below?**