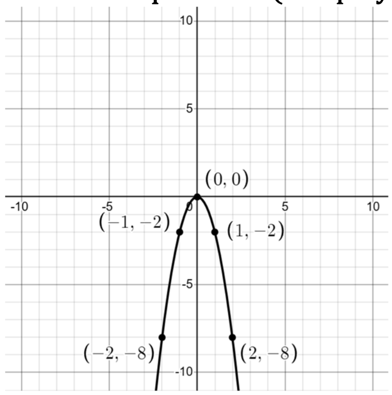
PC12 Polynomial Functions Lesson

- Polynomials functions and equations
- Factoring, including the factor theorem and the remainder theorem
- Graphing and the characteristics of a graph (e.g., degree, extrema, zeros, end-behaviour)
- Solving equations algebraically and graphically

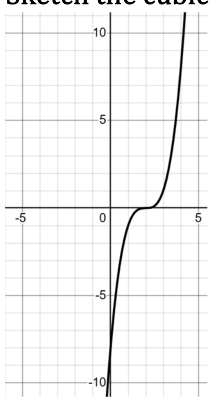
1. What is a polynomial function?
   In the form $ax^n + bx^{n-1} + \cdots$
   The coefficient of x are real numbers ex. $3, \frac{2}{3}, \sqrt{2}, \pi$
   But $n \in \mathbb{W}$ for exponents

2. Polynomial functions are smooth and c________________.
   continuous

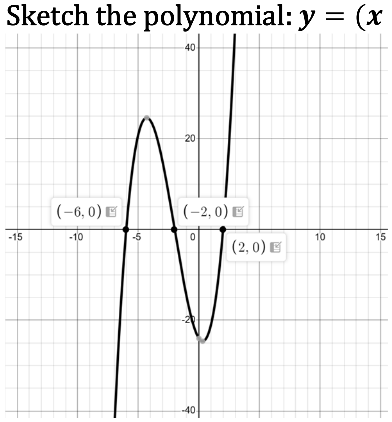
3. Sketch the quadratic (and polynomial) $f(x) = -2x^2$ and label 3 points.



4. Sketch the cubic polynomial $f(x) = (x-2)^3$ and label 3 points.



5. Sketch the polynomial: $y = (x-2)(x+6)(x+2)$



6. Is $P(x) = \pi x^5 - \sqrt{2}x^2 + 1.\overline{3}$ a polynomial function?
Yes

7. Is $P(x) = x^3 - 2x + \frac{1}{x}$ a polynomial function?
No

8. $f(x) = (2x+1)(x-2)(x+4)^{\frac{2}{3}}$
   a. Find the x-intercepts.
   $2, -\frac{1}{2}, -4$

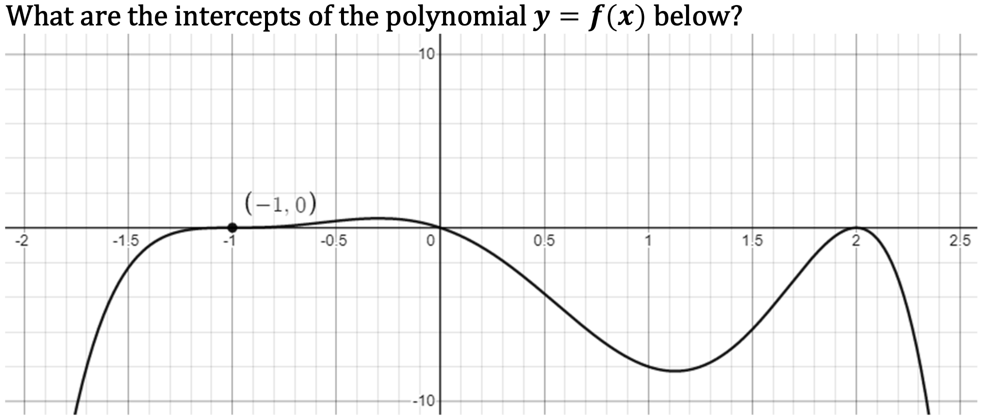
   b. Is $f(x)$ a polynomial?
   No (exponent is not a whole number)

9. Factor to find the roots of $2x^3 + x^2 - 6x$
$x(2x^2 + x - 6)$
$x(2x-3)(x+2)$
The roots are: $\frac{3}{2}, -2, 0$

10. Factor to solve when you land in the water $h(t) = 8 - 18t^2$
$0 = 2(4 - 9t^2)$
Divide by 2
$0 = 4 - 9t^2$
$9t^2 = 4$
$t^2 = \frac{4}{9}$
$t = \pm\sqrt{\frac{4}{9}} = \pm\frac{2}{3}$
$t = \frac{2}{3}$ (but reject negative time)

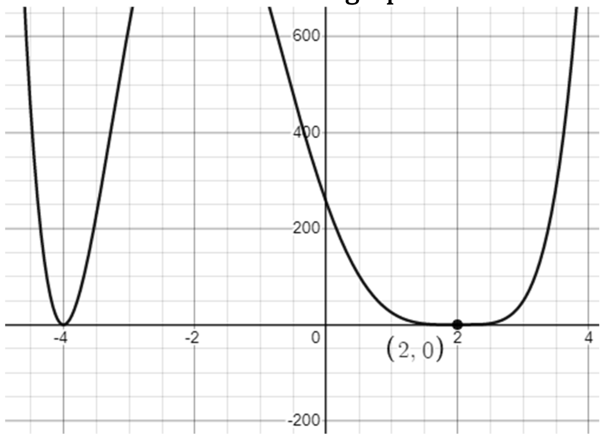
11. Given $P(x) = (3x-2)(x+\pi)^2$ what are the x-intercepts?
$\frac{2}{3}, -\pi$

12. What are the intercepts of the polynomial $y = f(x)$ below?



$-1, 0, 2$

13. What are the factors of the graph below?



$(x+4)$ and $(x-2)$ (note we're not sure what the exponent should be yet ...)

14. Solve $2x^2 = 4x$

$2x^2 - 4x = 0$

$2x(x-2) = 0$

$2, 0$

15. $P(x) = x(x-2)^2(x+4)^3(x-4)^2$

a. Evaluate $P(3)$

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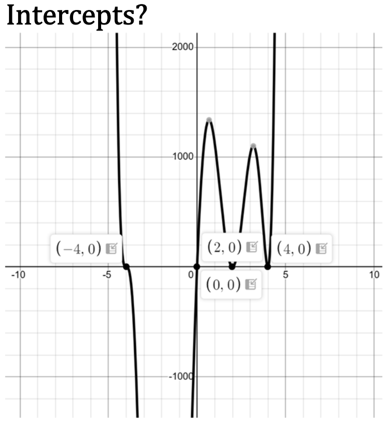
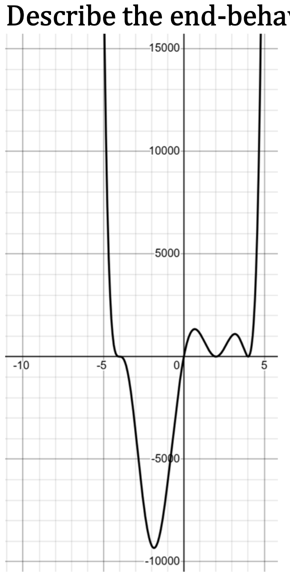
b. Intercepts?



x-ints: $0, 2, -4, 4$

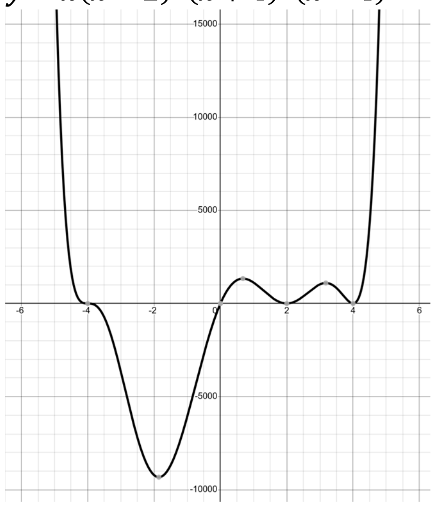
y-int (set $x = 0$): $P = 0(0-2)^2(0+4)^3(0-4)^2 = 0$

c. Describe the end-behavior of the graph as $x \to \infty$ and $x \to -\infty$

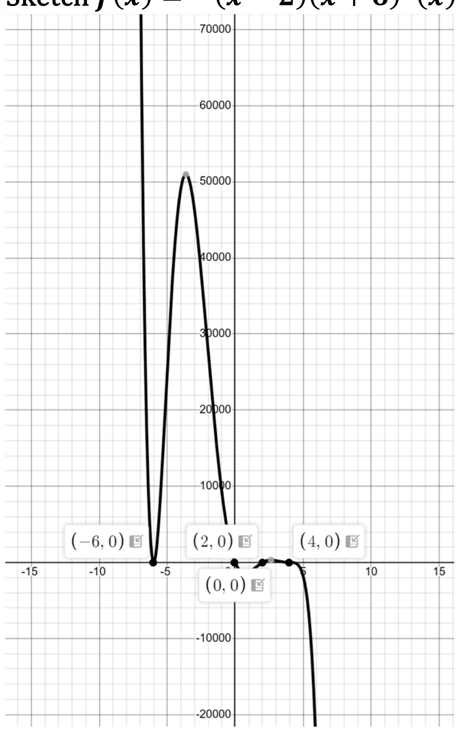


$+\infty$

d. Sketch the polynomial $P(x)$ while clearly showing the shape (based on the degree of the factors)
$y = x(x-2)^2(x+4)^3(x-4)^2$



16. Sketch $f(x) = -(x-2)(x+6)^2(x)(x-4)^3$



17. Identify the degree of the polynomial $P(x)$ below:
$P(x) = 16x^5 - 44x^6 + x^8 + 640x^4 - 512x^3 - 3072x^2 + 4096x$
8

18. Enrichment: What is the degree of the following polynomial: $y = 5x^2y^3 + 4x - 7$
5

19. $P(x) = (x-2)^2(x+6)^3$
What is the significance of the multiplicity (exponent of the factor) being 2?
Bounces of the x-axis.

20. What is the significance of the multiplicity (exponent of the factor) being 3 (or odd)?
Cuts through the x-axis with an "S" shape

21. $P(x) = (x-3)^1(x+2)^1x^1$
What is the significance of the multiplicity (exponent of the factor) being 1?
Cuts through the x-axis in relatively straight manner.

22. State the Factor Theorem
The Factor Theorem states that a polynomial $f(x)$ has a factor $(x-a)$ iff $f(a) = 0$.
Ex. $f(x) = (x-2)(x+4)$
$f(2) = 0$

23. State the Remainder Theorem
The Remainder Theorem states that if a polynomial $f(x)$ is divided by $(x-a)$, the remainder of this division is equal to $f(a)$. In other words, when you substitute $x = a$ into the polynomial, the value you get is the remainder when $f(x)$ is divided by $(x-a)$. This theorem is useful for evaluating polynomials at specific points and is closely related to the Factor Theorem.
$P(x) = x^3 + 2x^2 - x + 3$
According to the remainder theorem $P(2)$ gives you the remainder.
Rem = $(2)^3 + 2(2)^2 - (2) + 3 = 17$

$P(x) = x^2 - 2x - 8$
We will divide by $(x+2)$
$P(-2) = (-2)^2 - 2(-2) - 8 = 0$

By the way $P(x) = (x+2)(x-4) = 0$
$x = -2, 4$

24. State the rational root theorem (also called the Rational Zero Theorem)
The Rational Root Theorem states that if a polynomial $f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_0$ has a rational root of the form $\frac{p}{q}$, where p and q are integers with no common factors other than 1 (i.e., the fraction is in lowest terms), then:

- p must be a factor of the constant term a_0,
- and q must be a factor of the leading coefficient a_n.

This theorem helps identify possible rational roots of a polynomial, which can be tested by substitution or synthetic division to find actual roots.

25. $P(x) = x^3 + 2x^2 - 4x - 8.$

a. Factor by grouping

$x^2(x+2) - 4(x+2)$

$(x+2)(x^2-4)$

$(x+2)(x+2)(x-2)$

$P(x) = (x+2)^2(x-2)$

b. Factor using the rational root theorem and then sketch.

$P(2) = 0$

$\therefore (x-2)$ is a factor

Use long division: $(x^3 + 2x^2 - 4x - 8) \div (x-2)$

26. $P(x) = -x^4 + 6x^2 + 8x + 3.$ Fully factor and sketch this polynomial given $(x-3)$ is a factor.

27. Although you are welcome to use the Factor Theorem to factor polynomials, sometimes its faster to try to pull out a GCF first. Fully factor and sketch: $y = 4x^3 - 14x^2 + 12x$

28. $\frac{x^3-2x^2+3x+2}{x+2}$

a. Use long division to find the remainder.

b. Use synthetic division to find the remainder.

c. Use the remainder theorem to find the remainder.

d. Express in the form Quotient $+\frac{\text{remainder}}{\text{divisor}}$

29. $P(x) = \frac{x^3-2x+3}{x-1}$. Find the remainder using long division or synthetic division

30. Factor and sketch: $y = 2x^3 + x^2 - 4x - 3$

31. Factor and sketch: $y = x^4 - 2x^3 - 32x^2 + 96x$

32. Enrichment: $\boldsymbol{P(x) = (2x-3)^2(3x-1)}$

a. Identify the roots of this polynomials and sketch.

b. Expand this polynomial.

c. Factor this polynomial using the Rational Root Theorem.

33. Fractional roots: Factor $12x^4 - 20x^3 + 11x^2 - 2x$

34. What is the equation of the mystery degree 4 function below?