Function Transformations: Part I

- Vertical and horizontal translations, stretches, and reflections

1. $f(x) = 2x$. Describe the following transformations:
   a. $y = 3f(x)$

   b. $y = f(x) + 3$

   c. $y = f(x) - 1$

   d. $y = f(x - 4)$

   e. $y = f\left(x + \frac{3}{2}\right)$

   f. $y = f(x - 1) + 5$

   g. $y = -\frac{1}{2}f(x)$

   h. $y = f(-2x)$

2. In general, functions are transformed by the following parameters:
   $g(x) = af\big(b(x \pm c)\big) \pm d$
   Describe the transformational effect of these parameters

3. $f(x) = x^2$. Describe the following transformations:
   a. $y = f(x) - 4$

   b. $y = f(x) + 2$

   c. $y = f(x - 3)$

   d. $y = f(x + 1)$

   e. $y = f(x - 2) - 3$

f. $y = 2f(x)$

g. $y = -f(x)$

h. $y = \frac{1}{2}f(x-2)+5$

i. $y = f(2x)$

j. $y = f(2x-6)$

k. $y = f(-x)$

l. $y = \frac{2}{3}f\left(\frac{x}{2}+4\right)-1$

m. $y = -2f\left(4-\frac{3x}{4}\right)+3$

4. $f(x) = 2x+3$. Given $g(x) = f(x)-2$, find the actual equation of $g(x)$

5. $f(x) = x^2$. $g(x) = -2f(x+1)-3$. Find the actual equation of $g(x)$

6. $f(x) = x^2+2x$. $h(x) = \frac{1}{2}f(x-1)$. Find the actual equation of $h(x)$

7. $f(x) = 2x^2-3x+1$. $g(x) = -2f(2x-6)+3$. Find the actual equation of $g(x)$.

8. $f(x) = \log_2(x+2) - \frac{3^x}{e^{\sin x}}$. $g(x) = f(x-5)$. Find the actual equation of $g(x)$.

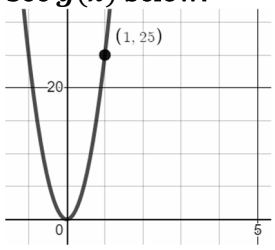
9. $h(t) = 3t^2 - t + 1$. $g(t) = -2h(1 - 3t) - h(2t) + 5$.
   a. Find the actual question of $g(t)$.

   b. Evaluate $h(-1)$

10. Classic function substitution problem: $f(x) = x^2$. $g(x) = \frac{f(x+h)-f(x)}{(x+h)-x}$. Actual equation of $g(x)$?

11. $f(x) = x^2$. $g(x) = 9x^2$. $h(x) = f(3x)$. Show that $h(x) = g(x)$.

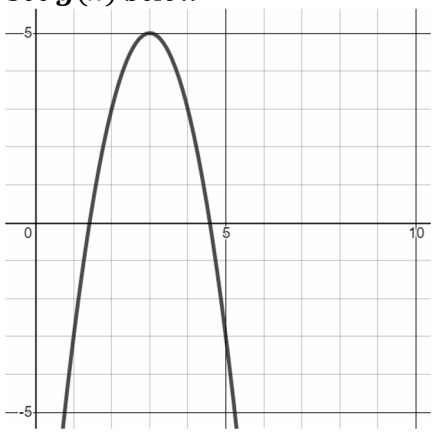
12. See $g(x)$ below:



   a. Describe $g(x)$ as a vertical transformation of $f(x) = x^2$

   b. Describe $g(x)$ as a horizontal transformation of $f(x) = x^2$

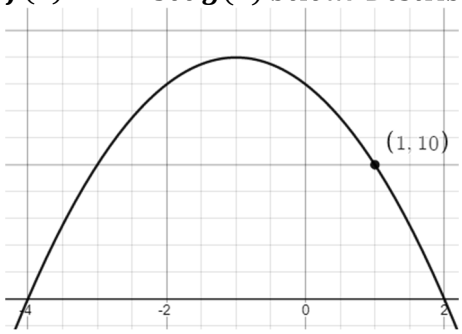
13. See $g(x)$ below



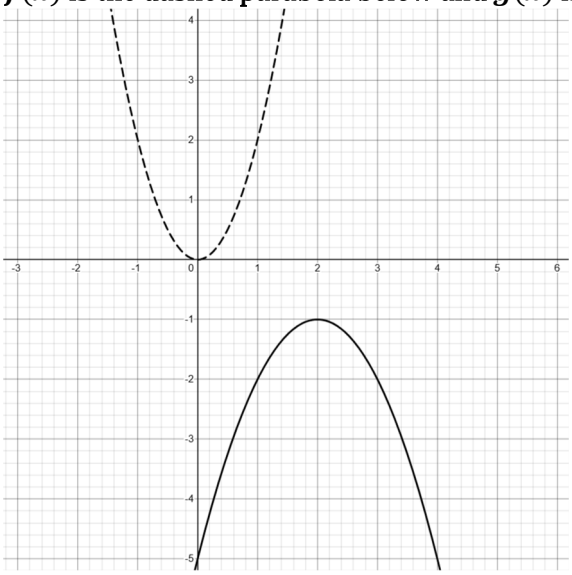
a. What is the equation of $g(x)$?

b. Describe $g(x)$ as a transformation of $f(x) = x^2$

14. $f(x) = x^2$. See $g(x)$ below. Describe $g(x)$ as a transformation of $f(x)$.



15. $f(x)$ is the dashed parabola below and $g(x)$ is the solid parabola:



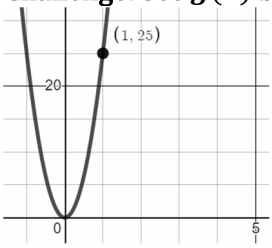
a. Find the equation $y = f(x)$.

b. Find the equation $y = g(x)$.

c. Describe $g(x)$ as a transformation of $f(x)$.

16. Challenge: $f(x) = x^2$. $g(x) = 5f(x)$. $h(x) = g(x) = f(bx)$. Find the possible values of b.

17. Challenge: See $g(x)$ below:



$f(x) = x^2$. See $g(x)$ in the diagram above.
Now describe $h(x)$ as a combination of both a vertical and horizontal transformation of $g(x)$