Implicit Differentiation (DO NOT WRITE ON)

Implicit differentiation is important because it allows us to find the derivative of a function when it's not explicitly solved for one variable in terms of another. In many real-world scenarios, relationships between variables are given as equations where isolating one variable isn't straightforward or even possible. With implicit differentiation, we can compute the derivative without having to solve for y explicitly, which can save time and effort – or be the only option when explicit solving isn't feasible.

- Higher order, implicit

1. Find y' for the following function: $x^2 + y^2 = 9$

2. Find the equation of the tangent line to: $x^2 + y^2 = 9$ at the point $(2, \sqrt{5})$

3. Given $9x^2 + 16y^2 = 144$, find the equation of the normal line at $x = 3$ given $y > 0$.

4. Find y': $x^3y^5 + 3x = 8y^3 + 1$

5. Find y': $x^2 \tan(y) + y^{10} \sec(x) = 2x$

6. Find y': $e^{2x+3y} = x^2 - \ln(xy^3)$

7. Assume that $x = x(t)$ and $y = y(t)$ and differentiate the following equation with respect to t. $x^3y^6 + e^{1-x} - \cos(5y) = y^2$

8. Use your understanding of logarithms and implicit differentiation to show that $\frac{d}{dx}a^x = \frac{a^x}{\ln a}$

9. Use logarithmic differentiation on $y = x^{2x}$ to find y'

Challenge

10. Justify that $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$ using implicit differentiation

11. Show that $\frac{d}{dx}\sin^{-1}\left(\frac{x}{k}\right) = \frac{1}{\sqrt{k^2-x^2}}$ using implicit differentiation

12. Show that $\frac{d}{dx}\frac{1}{k}\tan^{-1}\left(\frac{x}{k}\right) = \frac{1}{k^2+x^2}$