Math 9 Lesson 5: Proportional Reasoning

- Spatial proportional reasoning
- Scale diagrams, similar triangles and polygons, linear unit conversions
- Limited to metric units
- Drawing a diagram to scale that represents an enlargement or reduction of a given 2D shape
- Solving a scale diagram problem by applying the properties of similar triangles, including measurements
- Integration of scale for First Peoples mural work, use of traditional design in current First Peoples fashion design, use of similar triangles to create longhouses / models

1. How many cm in 2 meters?
   200 cm

2. How many mm in a km?

   $\mathbf{1\ km = 1000\ m}$
   $\mathbf{1000\ m = 100,000\ cm\ (1\ m = 100\ cm)}$
   $\mathbf{= 1,000,000\ mm}$

3. How many inches in a mile given 1 mile $= \mathbf{5280}$ feet?

   $\mathbf{1\ mile = 5280\ feet}$
   $\mathbf{5280 \times 12 = 63,360\ in}$

4. If a 10 cm long toy car is at a scale of 1:20 how long is the car in real life in meters?

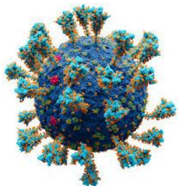
   The toy is smaller than the real car
   To go from a toy to the real car we multiply by 20
   $\mathbf{10\ cm \times 20 = 200\ cm = 2\ m}$

5. A right triangle has side lengths of 5-12-13.
   This triangle is enlarged by a factor of 2.
   a. Sketch the larger triangle
      $\mathbf{10 - 24 - 26}$

   b. What is the perimeter of the larger triangle?
      $\boldsymbol{P = 10 + 24 + 26 = 60}$

   c. How many times larger is the area of the larger triangle vs. the smaller triangle?
      $A_{\text{small}\Delta} = \frac{(12)(5)}{2} = 30$
      $A_{\text{large}\Delta} = \frac{(24)(10)}{2} = 120$
      $\text{Times larger} = \frac{120}{30} = 4$

6. A picture of the virus is 5 cm long. If the virus is 100 nanometers ("billionth") long in real life, what is the scale of this picture? (ex. 200:1, 1:2000, etc.)



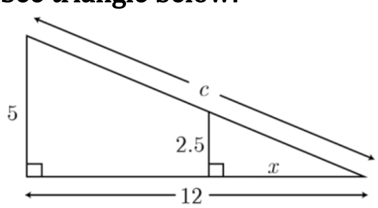
$\mathbf{500,000:1}$

7. The following toy car is 6 cm wide:

If the real racing car is 2 m wide, what is the scale factor?
6 cm : 2 m
6 cm : 200 cm
6 : 200
$\mathbf{1:33.\overline{3}}$

8. Your toy plane is 5 cm wide. It is at a scale of $2:170$. How large is the plane in real life?
$\mathbf{5\times\frac{170}{2}\approx 425\ cm\ or\ 4.25\ m}$

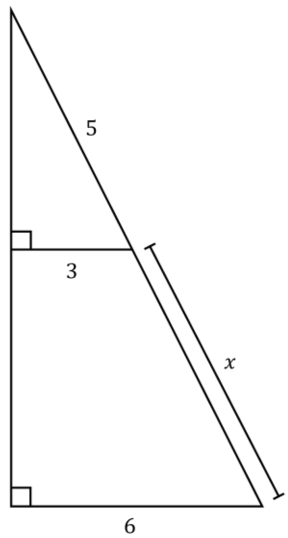
9. See triangle below:



a. Find c
13

b. Find x
6

10. Find x in the diagram below:



$\frac{5}{3} = \frac{5+x}{6}$

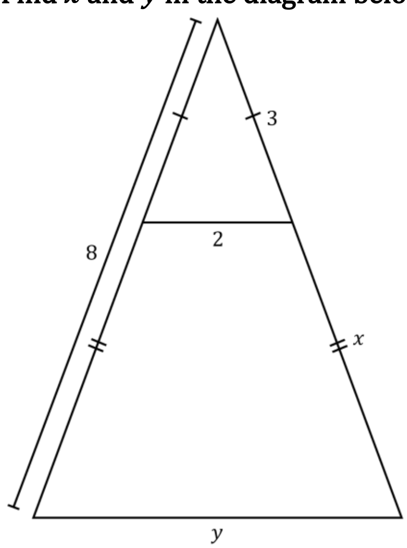
$3(5 + x) = 30$

$15 + 3x = 30$

$3x = 15$

$x = 5$

11. Find x and y in the diagram below:



$x = 8 - 3 = 5$

$\frac{3}{2} = \frac{8}{y}$

$3y = 16$

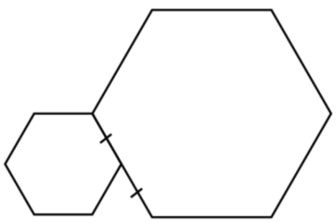
$y = \frac{16}{3}$

12. The Burj Khalifa is about 830 m tall. The ruler measurement of a picture of this building is 6 cm. The ruler measurement of the picture of the monster is 2 cm.
    a. How tall is the monster in real life?

$$\frac{830}{6} = \frac{x}{2}$$
$$6x = 1660$$
$$x = \frac{1660}{6} = \frac{830}{3}\text{ m}$$

    b. What assumption are you making when estimating the height of the monster?
    The monster is beside the building (not in front).

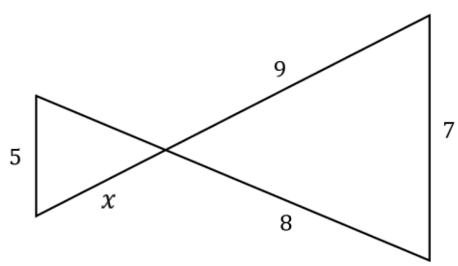
13. The perimeter of the small hexagon in the diagram below is 12 m.



    a. Find the side length of the large hexagon.
    Length of small is 2 m per side
    Length of large side is 4 m

    b. Express the perimeter of the smaller hexagon to the perimeter of the larger hexagon as a simplified ratio.
    $12 : 24 \rightarrow 1 : 2$

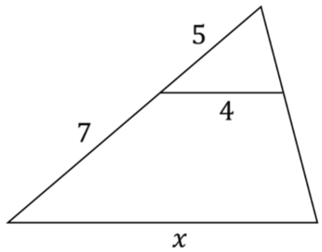
14. Find x in the diagram below:



$$\frac{x}{5} = \frac{9}{7}$$

$\boldsymbol{7x = 45}$
$\boldsymbol{x = \frac{45}{7}}$

15. See diagram below:

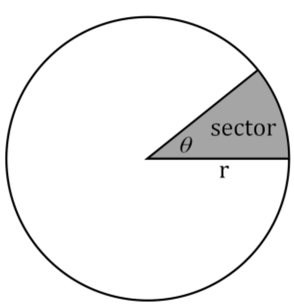


a. Under what conditions are the two triangles similar?
Line 4 and line x are parallel

b. Find x
$\boldsymbol{\frac{x}{4} = \frac{12}{5}}$
$\boldsymbol{5x = 48}$
$\boldsymbol{x = \frac{48}{5}}$

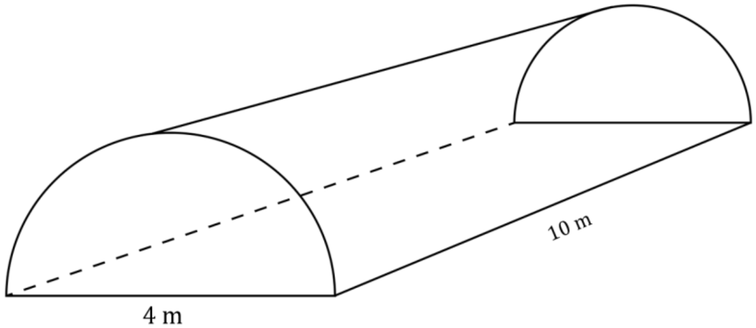
16. Challenge 1:
360 degrees is equal to 2π radians. The formula for the circumference of a circle is $C = 2\pi r$ and the area of a circle is $A = \pi r^2$. Show that the arc length of a sector of a circle is $arc = \theta r$.
$\boldsymbol{\frac{\theta}{2\pi} = \frac{\text{arc}}{2\pi r}}$
$\boldsymbol{\text{arc} = \frac{2\pi r\theta}{2\pi} = r\theta}$

17. Challenge 2:
Why is the area of the sector below $A_{sector} = \frac{\theta r^2}{2}$?



$\boldsymbol{\frac{\theta}{A_{\text{sector}}} = \frac{2\pi}{\pi r^2}}$
$\boldsymbol{2\pi \cdot A_{\text{sector}} = \theta \cdot \pi r^2}$
$\boldsymbol{A_{\text{sector}} = \frac{\theta \cdot \pi r^2}{2\pi} = \frac{\theta r^2}{2}}$

18. Challenge 3:
See the Pit House below:



a. How many times does the volume of the Pit House grow by doubling the dimensions?
8 times

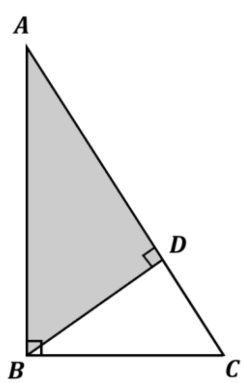
b. How many times does the area of the Pit House grow by doubling the dimensions?
4 times

c. How large does the Pit House's area scale up by increasing the dimensions by a factor of n?
n^2

d. How large does the Pit House's volume scale up by increasing the dimensions by a factor of n?
n^3

e. Does this scaling ratio increase for all types of shapes?
Yes

19. See below:



In $\Delta ABC, AB = 24$ and $BC = 10$. BD is $\perp$ to AC.
Find the ratio of the shaded area to the unshaded area.
Length AC is found using the Pythagorean Theorem
$(AC)^2 = 24^2 + 10^2$
$AC = \sqrt{24^2 + 10^2} = 26$
Using similar triangles:
$\frac{10}{26} = \frac{BD}{24}$
$26\ BD = 240 \rightarrow BD = \frac{120}{13}$
$(AD)^2 = (AB)^2 - (BD)^2$

$$AD = \sqrt{24^2 - \left(\frac{120}{13}\right)^2} = \frac{288}{13}$$

$$A_{\text{shaded}} = \frac{1}{2} \cdot BD \cdot AD = \frac{1}{2}\left(\frac{120}{13}\right)\left(\frac{288}{13}\right) = \frac{17280}{169} \approx 102.24852$$

$$CD = AC - AD = 26 - \frac{288}{13} = \frac{50}{13}$$

$$A_{\text{white}} = \frac{1}{2} \cdot CD \cdot BD = \frac{1}{2} \cdot \frac{50}{13} \cdot \frac{120}{13} = \frac{3000}{169} \approx 17.75147$$

Ratio of shaded : unshaded is

$$\frac{17280}{169} \div \frac{3000}{169} = \frac{144}{25}$$

144: 25 or 5.76: 1