

## TOPIC B: POWERS WITH RATIONAL EXPONENTS

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Powers is another topic in the BC Pre-Calculus 11 curriculum. Although you may have learned about exponent laws since Grade 9, this is the first year that BC math students fully learn the topic of powers which include negative exponents and fractional exponents.

- Positive and negative rational exponents
- Exponent laws
- Evaluation using order of operations
- Numerical and variable bases

1.  $2^{10}$   
 $1024$

2.  $(-2)^4$   
 $16$

3.  $-2^4$   
 $-16$

4.  $0^1 + 1^0$   
 $0 + 1 = 1$

5. Enrichment:  $0^0$   
Undefined

6.  $-3(-3)^2$   
 $-3 \times 9 = -27$

7.  $(-1)^{1234}$   
 $1$

8.  $3\left(\frac{2}{3}\right)^2$   
 $3 \times \frac{4}{9} = \frac{4}{3}$

9.  $2^{-3}$   
 $\frac{1}{2^3} = \frac{1}{8}$

10.  $\left(\frac{3}{5}\right)^{-3}$   
 $\left(\frac{5}{3}\right)^3 = \frac{125}{27}$

11.  $\frac{a^3}{a^{-2}}$   
 $a^5$

$$12. \frac{x(x^5)^2(x^3)}{x^{-1}}$$

$$x \times x^{10} \times x^3 \times x^1 = x^{15}$$

$$13. \left(\frac{8a^3b^5}{4a^2b^7}\right)^{-3}$$

$$\left(\frac{2a}{b^2}\right)^{-3} = \left(\frac{b^2}{2a}\right)^3 = \frac{b^6}{8a^3}$$

$$14. \frac{2x^2y(3xy)^{-3}}{(4xy^{-3})^2}$$

$$\frac{2x^2y}{16x^2y^{-6}(3xy)^3}$$

$$\frac{1y^7}{8 \times 27x^3y^3}$$

$$\frac{y^4}{216x^3}$$

$$15. x^{\frac{a}{b}} = \sqrt[b]{x^a}$$

$$16. \text{ Evaluate } 9^{\frac{1}{2}}$$

$$\sqrt{9} = 3$$

$$17. \text{ Evaluate } (0.09)^{1/2}$$

$$\sqrt{0.09} = 0.3$$

$$18. \text{ Evaluate } (-8)^{\frac{1}{3}}$$

$$\sqrt[3]{-8} = -2$$

$$19. \sqrt[3]{x} = x^k. \text{ Find } k$$

$$(x^{1/2})^{1/3} = x^{1/6}$$

$$k = 1/6$$

20. True or False:

a.  $\sqrt{9} = \pm 3$

False:  $\sqrt{9} = 3$

b. Given  $x^2 = 9, x = \pm 3$

True

$$21. 125^{-\frac{2}{3}}$$

$$(5^3)^{-\frac{2}{3}} = 5^{-2} = \frac{1}{25}$$

$$22. \text{ Convert } \sqrt{8} \text{ to a mixed radical}$$

$$2\sqrt{2}$$

$$23. \text{ Convert } \sqrt[3]{243} \text{ to a mixed radical}$$

$$\sqrt[3]{3 \times 3 \times 3 \times 3 \times 3}$$

$$3\sqrt[3]{9}$$

24. Convert  $3\sqrt{2}$  to an entire radical

$$\sqrt{2 \times 3 \times 3} = \sqrt{18}$$

25. Convert  $2\sqrt[5]{3}$  to an entire radical

$$\sqrt[5]{3 \times 2 \times 2 \times 2 \times 2} \\ \sqrt[5]{96}$$

26. Simplify  $x^2 \times x^{1/2}$  and express as a mixed radical in the form  $a\sqrt{b}$

$$x^{5/2} = \sqrt{x \times x \times x \times x} = x^2\sqrt{x}$$

27. Simplify  $\frac{x}{x^{2/3}}$ . Express your answer in the form  $x^{\frac{a}{b}}$

$$x^{1/3} \text{ (subtract exponents)}$$

28. Simplify  $(\sqrt[3]{x^2})(\sqrt[4]{x^5})$  using fractional exponents:  $x^{a/b}$

$$\left(x^{\frac{2}{3}}\right)\left(x^{\frac{5}{4}}\right) = x^{23/12}$$

29. Show that  $1^{-2.5} = 1$

$$LS = \frac{1}{1^{2.5}} = \frac{1}{1^{5/2}} = \frac{1}{\sqrt{1^5}} = \frac{1}{\sqrt{1}} = \frac{1}{1} = 1 = RS$$

30. Write  $\frac{15^6}{3^6}$  as a single power

$$\frac{(3 \times 5)^6}{3^6} = \frac{3^6 \times 5^6}{3^6} = 5^6$$

31. Solve  $\frac{3^{10}}{3^x} = 3^6$

$$3^{10-x} = 3^6$$

$$10 - x = 6$$

$$4 = x$$

32. Solve  $2^{x+1} = 8^{3-2x}$

$$2^{x+1} = (2^3)^{3-2x}$$

$$x + 1 = 9 - 6x$$

$$7x = 8$$

$$x = 8/7$$

33. Solve  $x^{3/5} = 2$

$$\left(x^{\frac{3}{5}}\right)^{\frac{5}{3}} = 2^{\frac{5}{3}}$$

$$x = 2^{\frac{5}{3}} \text{ or } \sqrt[3]{2^5} = 2\sqrt[3]{4}$$

34. Solve  $\left(\frac{2^{1-2x}}{2^{x+3}}\right)^3 = 4$

$$\text{Subtract exponents: } 1 - 2x - (x + 3) = -3x - 2$$

$$(2^{-3x-2})^3 = 2^2$$

$$2^{-9x-6} = 2^2$$

$$-9x - 6 = 2$$

$$-8 = 9x$$

$$-\frac{8}{9} = x$$

35. Simplify  $\frac{(0.6x^{-1})^{-2}}{\left(\frac{2}{x}\right)^3}$

$$\frac{\left(\frac{6}{10x}\right)^{-2}}{\frac{8}{x^3}}$$

$$\left(\frac{10x}{6}\right)^2 \div \frac{8}{x^3}$$

$$\left(\frac{5x}{3}\right)^2 \times \frac{x^3}{8}$$

$$\frac{25}{9} \times \frac{x^3}{8}$$

$$\frac{25}{72}x^3$$

36. Evaluate  $\frac{(-2)^{100}}{-2^{97}} + 8^{-\frac{1}{3}} \div 2$

$$-2^3 + \frac{1}{\sqrt[3]{8}} \div 2$$

$$-8 + \frac{1}{2} \times \frac{1}{2}$$

$$-8 + \frac{1}{4}$$

$$-\frac{32}{4} + \frac{1}{4} = -\frac{31}{4}$$

37. Simplify  $\left(\frac{-16x^{-2}y}{2x^{-3}y^3}\right)^{-2/3}$

using positive fractional exponents.

$$\left(\frac{-8x}{y^2}\right)^{-2/3} = \left(\frac{y^2}{(-2)^3x}\right)^{2/3}$$

$$\frac{y^{4/3}}{4x^{2/3}}$$

38. Challenge: Show that  $\sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a$

$$LS = x^{a/b}$$

$$RS = \left(x^{1/b}\right)^a = x^{a/b} = LS$$