

## TOPIC C: RADICALS

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Radicals is a math topic that has a connection with exponents. A fractional exponent can be rewritten as a radical. This year in Pre-Calculus 11 we will focus on simplifying radicals and working with radical equations. Next year you will learn more about graphing and transforming radical functions.

- Simplifying radicals
- Ordering a set of irrational numbers
- Performing operations with radicals
- Solving simple (one radical only) equations algebraically and graphically
- Identifying domain restrictions and extraneous roots of radical equations

1. Evaluate  $\sqrt{25}$

5

2. Solve  $x^2 = 25$

$\pm 5$

3. Express  $\sqrt{8}$  as a mixed radical

$\sqrt{2 \times 2 \times 2} = 2\sqrt{2}$

4. Express  $3\sqrt{3}$  as an entire radical

$\sqrt{3 \times 3 \times 3} = \sqrt{27}$

5. Express  $-2\sqrt[3]{3}$  as an entire radical

$= -\sqrt[3]{3 \times 2 \times 2 \times 2} = -\sqrt[3]{24}$  or  $\sqrt[3]{-24}$

6.  $\sqrt{-9}$

Undefined

7.  $\sqrt[3]{-8}$

-2

8.  $\sqrt{4\,000\,000}$

$\sqrt{2000 \times 2000} = 2000$

9.  $\sqrt{0.25}$

$\sqrt{0.5 \times 0.5} = 0.5$

10.  $\sqrt{\frac{4}{9}}$

$= \frac{\sqrt{4}}{\sqrt{9}}$

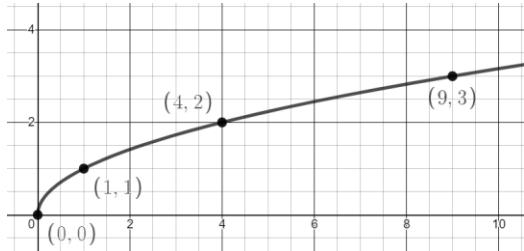
$= 2/3$

Or  $\sqrt{\frac{4}{9}} = \sqrt{\frac{2}{3} \times \frac{2}{3}}$

11. Order from least to greatest:  $\sqrt{9}, 2\sqrt{3}, \sqrt{30}, \pi$   
 $\sqrt{9}, \pi, 2\sqrt{3}, \sqrt{30}$

12.  $f(x) = \sqrt{x}$

- a. Sketch and label 3 points

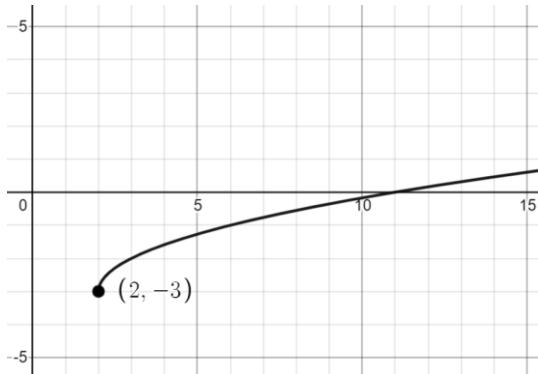


- b. Evaluate  $f(25)$

5

13.  $y = \sqrt{x-2} - 3$

- a. Sketch



- b. Domain?

$x \geq 2$

- c. Range?

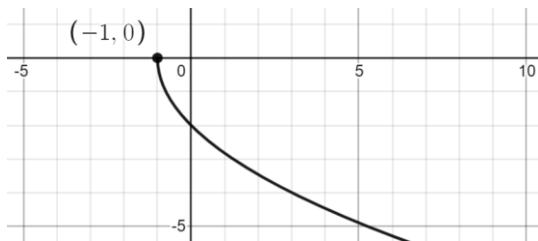
$y \geq -3$

14.  $y = \sqrt{x-a} + b$ . Given  $a, b > 0$ , describe the transformation.

$a$  units right,  $b$  units up.

15.  $y = -2\sqrt{x+1}$

- a. Sketch



- b. Domain?

$x \geq -1$

c. Range?

$$y \leq 0$$

16.  $y = a\sqrt{x+b} + c$ . Given  $a, b, c > 0$  describe the transformation.

Compared to  $y = \sqrt{x}$ , the graph is stretched vertically by a factor of  $a$ , shifted  $b$  units left and  $c$  units up.

17. Find the domain of  $y = \sqrt{x-3}$

$$x - 3 \geq 0$$

$$x \geq 3$$

18. Find the domain of  $\sqrt{3-5x}$

$$3 - 5x \geq 0$$

$$3 \geq 5x$$

$$\frac{3}{5} \geq x \text{ or } x \leq \frac{3}{5}$$

19. Find the domain of  $\frac{\sqrt{1-2x}}{x}$

$$1 - 2x \geq 0 \rightarrow 1 \geq 2x \rightarrow x \leq \frac{1}{2} \text{ and } x \neq 0$$

20. Find the domain of  $\frac{\sqrt{3x-2}}{x^2-9}$

$$x^2 - 9 \neq 0 \rightarrow x \neq \pm 3$$

$$3x - 2 \geq 0 \rightarrow 3x \geq 2 \rightarrow x \geq \frac{2}{3}$$

Combining these constraints we get:  $x \geq \frac{2}{3}$  and  $x \neq \pm 3$

21. Find the domain of  $\frac{2\sqrt{x}}{x^2+x-20}$

$$\frac{2\sqrt{x}}{(x+5)(x-4)}$$

$x \geq 0$  and  $x \neq -5, 4$

22. Rationalize:

a.  $\frac{1}{\sqrt{2}}$

$$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

b.  $\frac{4}{\sqrt{8}}$

$$\frac{4}{\sqrt{8}} \left( \frac{\sqrt{8}}{\sqrt{8}} \right) = \frac{4\sqrt{8}}{8} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

c.  $\frac{9}{6-\sqrt{3}}$

$$= \frac{9(6+\sqrt{3})}{(6-\sqrt{3})(6+\sqrt{3})} = \frac{9(6+\sqrt{3})}{36-3} = \frac{9(6+\sqrt{3})}{33} = \frac{3(6+\sqrt{3})}{11} \text{ or } \frac{18+3\sqrt{3}}{11}$$

$$\begin{aligned}
 \text{d. } & \frac{1}{\sqrt[3]{3}} \\
 & = \frac{1}{\sqrt[3]{3}} \times \frac{\sqrt[3]{3}}{\sqrt[3]{3}} \times \frac{\sqrt[3]{3}}{\sqrt[3]{3}} = \frac{\sqrt[3]{9}}{3} \\
 & \text{or } \frac{1}{3^{1/3}} = \frac{1}{3^{1/3}} \times \frac{3^{2/3}}{3^{2/3}} = \frac{\sqrt[3]{9}}{3}
 \end{aligned}$$

$$\begin{aligned}
 23. \text{ Simplify } & \sqrt{8} + 3\sqrt{2} \\
 & = 2\sqrt{2} + 3\sqrt{2} \\
 & = 5\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 24. \text{ Simplify } & \sqrt{8} - \sqrt[3]{32} + 3\sqrt{2} + \sqrt[3]{4} \\
 & = 2\sqrt{2} + 3\sqrt{2} - \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2} + \sqrt[3]{4} \\
 & = 5\sqrt{2} - 2\sqrt[3]{4} + \sqrt[3]{4} \\
 & = 5\sqrt{2} - \sqrt[3]{4}
 \end{aligned}$$

$$\begin{aligned}
 25. \text{ Simplify } & \frac{\sqrt{12}}{2} \\
 & = \frac{\sqrt{2 \times 2 \times 3}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 26. \text{ Evaluate } & \sqrt{\frac{9}{25}} \\
 & = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}
 \end{aligned}$$

$$\begin{aligned}
 27. \text{ Simplify } & \frac{-2+\sqrt{12}}{-2} \\
 & = \frac{-2+\sqrt{2 \times 2 \times 3}}{-2} = \frac{-2+2\sqrt{3}}{-2} = -\frac{2}{-2} + \frac{2\sqrt{3}}{-2} = 1 - \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 28. \text{ Simplify } & 2\sqrt{3} \times 3\sqrt{2} \\
 & = 6\sqrt{6}
 \end{aligned}$$

29. Expand and simplify:

$$\begin{aligned}
 \text{a. } & 2\sqrt{2}(\sqrt{4} - 3\sqrt{2} + 1) \\
 & 2\sqrt{8} - 6(2) + 2\sqrt{2} = 2 \times 2\sqrt{2} - 12 + 2\sqrt{2} \\
 & = 4\sqrt{2} + 2\sqrt{2} - 12 \\
 & = 6\sqrt{2} - 12
 \end{aligned}$$

$$\begin{aligned}
 \text{b. } & (2 - \sqrt{2})^2 \\
 & = (2 - \sqrt{2})(2 - \sqrt{2}) \\
 & = 4 - 2\sqrt{2} - 2\sqrt{2} + 2 \\
 & = 6 - 4\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{c. } & (\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2}) \\
 & 3 - 2 = 1
 \end{aligned}$$

$$\begin{aligned}
 \text{d. } & 3(\sqrt{8} - \sqrt{2})(1 - \sqrt{8}) \\
 & = 3(2\sqrt{2} - \sqrt{2})(1 - 2\sqrt{2})
 \end{aligned}$$

$$\begin{aligned}
&= 3(\sqrt{2})(1 - 2\sqrt{2}) \\
&= 3\sqrt{2}(1 - 2\sqrt{2}) \\
&= 3\sqrt{2} - 6(2) \\
&= 3\sqrt{2} - 12
\end{aligned}$$

$$\begin{aligned}
e. \quad &(\sqrt{8} - 1)^3 \\
&= (2\sqrt{2} - 1)(2\sqrt{2} - 1)^2 \\
&= (2\sqrt{2} - 1)(8 - 4\sqrt{2} + 1) \\
&= (2\sqrt{2} - 1)(9 - 4\sqrt{2}) \\
&= 18\sqrt{2} - 8(2) - 9 + 4\sqrt{2} \\
&= 22\sqrt{2} - 25
\end{aligned}$$

30. A rectangle has a base of  $4\sqrt{2} - 2\sqrt{3}$  and a height of  $\sqrt{8} - \sqrt{3}$

a. Area in simplified form?

$$\begin{aligned}
A &= (4\sqrt{2} - 2\sqrt{3})(\sqrt{8} - \sqrt{3}) \\
&= (4\sqrt{2} - 2\sqrt{3})(2\sqrt{2} - \sqrt{3}) \\
&= 8(2) - 4\sqrt{6} - 4\sqrt{6} + 2(3) \\
&= 16 - 8\sqrt{6} + 6 \\
&= 22 - 8\sqrt{6}
\end{aligned}$$

b. Perimeter in simplified form?

$$\begin{aligned}
P &= 2(4\sqrt{2} - 2\sqrt{3}) + 2(\sqrt{8} - \sqrt{3}) \\
&= 8\sqrt{2} - 4\sqrt{3} + 2(2\sqrt{2} - \sqrt{3}) \\
&= 8\sqrt{2} - 4\sqrt{3} + 4\sqrt{2} - 2\sqrt{3} \\
&= 12\sqrt{2} - 6\sqrt{3}
\end{aligned}$$

31. A cylinder has a diameter of  $\sqrt{8}$  and a height of 10

a. Volume?

$$\begin{aligned}
V &= \pi r^2 h \\
V &= \pi (\sqrt{2})^2 (10) = 20\pi \text{ units}^3
\end{aligned}$$

b. Area including the bottom?

$$\begin{aligned}
A &= 2\pi r^2 + \pi d h \\
A &= 2\pi (\sqrt{2})^2 + \pi \sqrt{8}(10) = 4\pi + 20\sqrt{2}\pi \text{ units}^2
\end{aligned}$$

32. Solve:

a.  $\sqrt{x} = 3$   
 $x = 9$

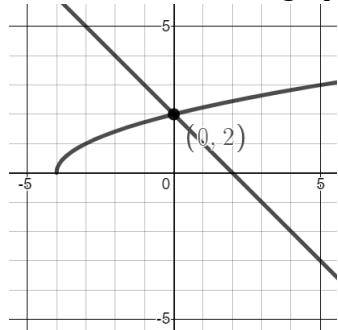
b.  $2\sqrt{x} = 4$   
 $\sqrt{x} = 2$   
 $x = 4$

c. Solve  $\sqrt{x - 2} = 3$   
Square both sides  
 $x - 2 = 9$

$$x = 11$$

33.  $\sqrt{x+4} = 2 - x$

- a. Estimate the solution graphically



$$x = 0 \\ (y = 2)$$

- b. Find the point of intersection algebraically

$$\sqrt{x+4} = 2 - x$$

Square both sides

$$x + 4 = (2 - x)^2$$

$$x + 4 = 4 - 4x + x^2$$

$$0 = x^2 - 5x$$

$$0 = (x)(x - 5)$$

$$0 = (x - 0)(x - 5)$$

$$x = 0 \text{ or } x = 5$$

$$\text{When } x = 0, y = 2$$

- c. Check for extraneous roots

Test  $x = 0$

$$LS = \sqrt{x+4} = \sqrt{0+4} = 2$$

$$RS = 2 - x = 2 - 0 = 2$$

$$LS = RS \therefore x = 0 \text{ is a solution}$$

Test  $x = 5$

$$LS = \sqrt{x+4} = \sqrt{5+4} = 3$$

$$RS = 2 - x = 2 - 5 = -3$$

$$LS \neq RS \therefore x = 5 \text{ is NOT a solution!}$$

Be careful about rejecting the extraneous solution.

- d. Find the point of intersection

$$\text{When } x = 0, y = 2 \rightarrow (0,2)$$

34. Solve  $\sqrt{x-1} = 2 - \frac{x}{2}$

Multiply both sides by 2

$$2\sqrt{x-1} = 4 - x$$

Square both sides:

$$(2\sqrt{x-1})^2 = (4-x)^2$$

$$4(x-1) = 16 - 8x + x^2$$

$$4x - 4 = 16 - 8x + x^2$$

Move terms to the right

$$0 = x^2 - 12x + 20$$

Factor

$$0 = (x-10)(x-2)$$

$$x = 10, 2$$

But we reject  $x = 10$  as an extraneous root.

When  $x = 10$  the left side of the equation  $LS = \sqrt{x-1} = \sqrt{10-1} = \sqrt{9} = 3$

$$\text{But the right side of the equation } RS = 2 - \frac{x}{2} = 2 - \frac{10}{2} = 2 - 5 = -3$$

$LS \neq RS$  therefore reject the solution  $x = 10$ .

Whereas when  $x = 2$ ,  $LS = RS$ .

35. Solve  $\sqrt{2x+2} + 3 = x$

Isolate the radical

$$\sqrt{2x+2} = x - 3$$

Now, square both sides

$$2x + 2 = (x-3)^2$$

$$2x + 2 = x^2 - 6x + 9$$

$$0 = x^2 - 8x + 7$$

$$0 = (x-7)(x-1)$$

Possible solutions:  $x = 7$  (reject  $x = 1$ )

Test  $x = 7$

$$LS = \sqrt{2x+2} + 3 = \sqrt{2(7)+2} + 3 = 4 + 3 = 7$$

$$RS = x = 7$$

$$LS = RS \therefore x = 7$$

Test  $x = 1$

$$LS = \sqrt{2x+2} + 3 = \sqrt{2(1)+2} + 3 = 5$$

$$RS = x = 1$$

$LS \neq RS \therefore x = 1$  is not a solution

36. Enrichment: Solve  $\sqrt{x+1} - 2 = \sqrt{x-3}$

Square both sides

$$(\sqrt{x+1} - 2)^2 = x - 3$$

$$x + 1 - 4\sqrt{x+1} + 4 = x - 3$$

$$8 = 4\sqrt{x+1}$$

Divide by 4

$$2 = \sqrt{x+1}$$

Square both sides again

$$4 = x + 1$$

$$x = 3$$

37. Enrichment: Simplify  $\sqrt{x^2}$

$$\sqrt{x^2} = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

